When Goliath Wins:
A Dynamic Theory of Firms in Asset Management

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Abstract

I develop an equilibrium theory of asset management firms as a nexus of relationships between investors and managers. Investors and managers can meet directly. But when either side departs, the remaining side must search again. In a firm, the remaining capital and managers can instead be reassigned to each other without new search. Anticipating such departures, investors and managers come together inside firms to reduce their need for future external search. This dynamic consideration makes firm value exhibit increasing returns to relationships. Estimated on U.S. data on firms' entry, size, and growth, the model attributes 61% of the value of asset management firms to reduced dependence on future external search.

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1 Introduction

Asset management solves the problem of allocating investor capital across assets. This happens in two steps: allocating capital across managers, and having those managers allocate capital across assets. This paper studies the first step, starting from the observation that much of this allocation happens inside firms rather than directly between investors and managers. Why are capital and managers matched inside firms, and what makes these firms valuable?

To answer these questions, I build a model in which an asset management firm is a nexus of relationships between investors and managers. Managers use the capital assigned to them to produce services for investors. Investors and managers can match directly. A match lasts until either side departs at random, at which time the remaining side must search again in the external market. Inside a firm, however, when some investors or managers depart, the remaining capital can be reassigned to the remaining managers without external search. Anticipating potential departures, investors and managers join firms to reduce their need for future search. Because a larger firm has more options to reassign after departures than two half-size firms do separately, firm value exhibits increasing returns to relationships.

I take the model to U.S. asset management data from Form ADV filings, estimating it on moments of entry, growth, and firm size. The estimates attribute 61% of firm value to reduced dependence on future external search, with 25 percentage points coming from increasing returns to relationships. The model attributes the remaining 39% of firm value to other sources, such as intangibles.

In the model, a firm's boundary is the set of investors and managers it can reassign without new search. Relationships form through directed search in the external market and are used in the internal market. Firms grow by posting offers to unmatched investors and managers, who in turn direct their search to the best offers. Unmatched managers can also post offers directly to unmatched investors, and a match there creates a small entrant firm. To grow or enter faster, a firm or manager must offer larger payments to attract more searchers. Firms therefore slow their growth, trading off the benefits of new recruits against the cost of attracting them quickly. In equilibrium, these offers and search decisions determine how fast firms grow, how fast new firms form, and how surplus is split among investors, managers, and firms.

I think of departure shocks as changes in investors' preferences or managers' capacity to serve the same investors. These shocks can leave a firm investor-heavy or manager-

heavy, meaning its ratio of investors to managers becomes unusually high or low. Because each manager produces services with diminishing returns, an investor-heavy firm and a manager-heavy firm could produce more with the same members if they could reassign capital and managers across their boundaries. While they search separately in the external market to repair their imbalances, they forgo this extra output. Having more relationships inside the same firm reduces both the search costs of repairing imbalances and the output forgone while imbalanced. Dynamics thus make firms valuable and generate increasing returns to relationships.

The theory yields several testable implications for the investor-to-manager ratio, which hold in the data. First, firms repair existing imbalances gradually. An investor-heavy firm grows its manager base faster than its investor base over multiple years, while a manager-heavy firm does the reverse. Second, the adjustments of two small firms imbalanced in opposite directions are offsetting, so their combined ratio evolves like that of a single large firm of the same total size. Third, inside a large firm, an imbalance in one part can offset an opposite imbalance in another. Therefore, at the same ratio, large firms' ratios are steadier going forward than those of half-size firms.

I estimate the model using standard firm dynamics moments, such as firm size and dispersion, and new moments on how quickly firms repair imbalances in their investor-to-manager ratios. I measure this repair over the next two years, separately for investor and manager bases. Search frictions slow this repair and can keep a firm far from its ideal ratio for multiple years. Together, these moments discipline search frictions and how much firms need to adjust.

Using the estimated model, I unpack the dynamic considerations that make relationships valuable. First, consider splitting a firm into single-manager partitions, with investors allocated across partitions so that the split costs no output today. Each partition can reassign its existing investors' capital to new managers hired as it grows again but cannot assign across partitions. The difference in value between the firm and its partitions measures increasing returns to relationships. It accounts for 25% of the firm's value. Then, consider further taking the partitions and not allowing them to reassign to new managers. The further value lost accounts for 36% of the firm's value. The ability to reassign across and within partitions is how dynamic considerations about relationships affect the firm's value. Therefore, 61% of the firm's value is due to these dynamic considerations. The remaining 39% comes from the current assignments of capital to managers, investor recruiting, and intangibles.

I then examine how the share of firm value from increasing returns to relationships

responds to technological change, proxied by lower search frictions and a lower output elasticity with respect to managers.[1] First, in partial equilibrium, lower search frictions reduce this share because avoiding future search becomes less important. In full equilibrium, however, each firm forms many more relationships due to reduced search frictions and gains more options to reassign inside its boundary. Therefore, the share rises as search frictions fall.

Second, consider a decline in the output elasticity with respect to managers, so that fewer managers are needed to produce the same output. In partial equilibrium, a large decline in this elasticity reduces the share of firm value from increasing returns to relationships. Because manager departures affect production less, firms need to search less quickly for new managers, and relationships formed today save less future search. In full equilibrium, however, firms' demand for investors rises relative to managers. This greater demand raises the cost of attracting investors quickly in the external market. A firm that becomes manager-heavy finds it costlier to repair its imbalance. As a result, avoiding future search becomes more important. In the estimated model, this equilibrium force is stronger, and thus increasing returns to relationships account for a larger share of firm value.

Although the model is written for asset management, its mechanism may apply more broadly to two-sided markets where human capital is the primary input. Law and consulting firms, for example, bring clients and professionals inside their boundaries to assign them to one another, and individual professionals can likewise found entrant firms on their own. A similar logic about dynamic considerations about relationships applies to two-sided matching platforms such as ridesharing, where platforms match riders and drivers internally while recruiting new riders and drivers takes time.[2]

The paper proceeds as follows. The next subsection reviews the related literature. Section 2 develops a model in which firms form relationships through search in the external market and later reassign capital and managers in their internal markets without new search. Section 3 presents the key theoretical results. Section 4 extends the baseline model, describes the data, and estimates the model. Section 5 quantifies the share of firm value that comes from reduced dependence on future external search, and counterfactuals examine how the share of firm value from increasing returns to relationships changes when

[1] For example, artificial intelligence may shorten search in the external market and enable firms to produce the same output with fewer managers.

[2] The equilibrium differs there, however, because a single driver cannot start a competing platform, whereas an individual manager can start an entrant asset management firm.

search frictions fall and when the output elasticity with respect to managers falls.

1.1 Literature Review

This paper contributes to the literature on delegated asset management and financial intermediation. It is closest to Berk and Green (2004) and Gârleanu and Pedersen (2018). In the canonical model of Berk and Green (2004), managerial skill is scarce and investor capital flows to it without frictions. Gârleanu and Pedersen (2018) model search frictions investors face in finding asset managers.[3] Neither distinguishes managers from the firms that employ them. The distinction between managers and firms is the focus of this paper.

Empirical work motivates the model's distinction between firms and their managers. Several papers document an important contribution of individual managers or partners to investment performance (Ewens and Rhodes-Kropf, 2015, Berk et al., 2017, Jackson, 2025). Empirical studies find that asset management firms reassign capital and managers internally and that this reassignment increases investment value added (Berk et al., 2017, Luo et al., 2024, Braun et al., 2026). Work on private equity also documents differences in firm scale, the allocation of partnership income toward founders, and the sale of stakes in firms' fee income (Metrick and Yasuda, 2010, Ivashina and Lerner, 2019, Gahng and Jackson, 2026).

This paper also contributes to the literature on the theory of the firm. Classic theories examine what distinguishes transactions inside a firm from market exchange and where the boundary between them lies (Coase, 1937, Williamson, 1985, Grossman and Hart, 1986, Holmström, 1999, Rajan and Zingales, 2001). Work on internal capital markets considers both their benefits and the investment distortions caused by incentive conflicts (Alchian, 1969, Stein, 1997, Rajan et al., 2000, Scharfstein and Stein, 2000). Empirical studies examine resource allocation and performance inside firms using reduced-form evidence and structural estimation, primarily in conglomerates (Lamont, 1997, Maksimovic and Phillips, 2002, Schoar, 2002, Seru, 2014, Matvos and Seru, 2014), including internal labor markets (Giroud and Mueller, 2015, Tate and Yang, 2015). I instead ask how valuable internal markets are for using relationships between investors and managers in the future. I develop a model of firm dynamics and quantify this value.

The paper also relates to the literature on firm dynamics. In Hopenhayn (1992) and Luttmer (2007), a firm is a productivity level that changes over time. In Klette and

[3] Sirri and Tufano (1998), Hortaçsu and Syverson (2004), and Roussanov et al. (2021) study how search frictions shape fund flows, fees, and marketing in mutual funds.

Kortum (2004), a firm is a collection of products. Here, a firm is a nexus of relationships between investors and managers. A firm grows by bringing in more investors and managers through search. In the quantification, I add intangibles that the firm owns, which work like productivity in Hopenhayn (1992) and Luttmer (2007). I then separate the value of relationships from other sources of firm value.

The paper also relates to search-based theories of financial intermediation (Duffie et al., 2005, Lagos and Rocheteau, 2009, Weill, 2020), where search leads to one-time trades rather than lasting matches between investors and managers. Its search structure builds on directed search (Moen, 1997) and multi-worker firm models (Lester, 2010, Hawkins, 2013, Acemoglu and Hawkins, 2014, Kaas and Kircher, 2015), with both sides searching, so that investors and managers can also find each other directly.

2 Model

This section develops an equilibrium model of asset management firms as a nexus of relationships between investors and managers. To sharpen the exposition, in the baseline model, firms differ only in the numbers of investors and managers they connect. Investors and managers can also meet directly, and each such match starts a new firm. The search decisions of investors, managers, and firms lead to an equilibrium.

2.1 Agents

Time is continuous, and the horizon is infinite. There are continua of investors and managers. Agents are risk-neutral, have transferable utility, and discount at rate $r > 0$.

The total mass of managers is normalized to 1, while the mass of investors is $I > 0$. Thus, I is the investor-manager population ratio. Each agent is born unmatched, and it takes time to meet with counterparties. Once agents match, they form long-term relationships that end only through exit.

Each manager has a technology to produce services, and each investor has one unit of (non-depreciating) capital that can be used in this production.

Investors and managers exit the capital and labor markets independently at Poisson rates $\epsilon_I > 0$ and $\epsilon_M > 0$, respectively. Each exiting agent is replaced by a new, unmatched agent of the same type, so population masses remain fixed. I interpret these exit shocks as changes in investors' preferences, desired allocations, and managers' capacity to serve the same investors, and more generally as reasons to reallocate capital and labor.

2.2 Technologies

There are three technologies: an internal market, a production function, and a search function. All firms possess the same technologies. Production and search are assumed to have constant returns to scale. Therefore, they do not favor larger firms on their own. I make these assumptions to emphasize the contrast with my later results, which show that firm value endogenously exhibits increasing returns to scale because of dynamic considerations about reassignment in internal markets.

The first technology is an internal market. An internal market connects a nexus of investors and managers in two ways. First, the members produce together and commit to transfers among themselves. Capital can be assigned to managers freely within the nexus. The nexus maximizes its joint surplus. Second, they recruit new members together, so a recruit whom one member meets joins the whole nexus.

I call a nexus connected by an internal market a "firm." A firm is summarized by $(i, m) \in \mathbb{Z}_+^2$, where i is the number of investors and m is the number of managers connected to the firm. I call these i investors the firm's investor base and these m managers its manager base.[4] When I refer to the firm and its policies, I mean its i investors and m managers and their joint policies. An investor or manager who is not in a firm is unmatched. An investor's capital is tied to the firm, not to particular managers of the firm. When a manager leaves, the capital he managed stays with the firm and goes to its other managers.

The second technology is production. Each manager produces $\tilde{f}(k)$ units of flow services from k units of investor capital. These services may, for example, be about investment performance, the net present value of the investments he makes today with the capital he manages. Investors may also demand asset management for other reasons, which $\tilde{f}$ can also represent. I assume $\tilde{f}$ is weakly increasing and concave, with $\tilde{f}(0) = 0 < \tilde{f}(1)$. Concavity means each manager has diminishing returns to the capital he manages (e.g., Berk and Green, 2004).

The firm internally assigns its investors' capital to its managers (see, for example, the empirical evidence from asset management in Berk et al., 2017, Braun et al., 2026).[5]

[4] A firm's investor base resembles customer capital (Gourio and Rudanko, 2014, He et al., 2024). Relative to this literature, firms here sit between two sides that can bypass them and search for each other directly.

[5] Assigning capital to managers does not necessarily require moving money across funds. In private equity, for example, firms choose which of their managers runs each deal and how much capital to assign to each, and these assignment decisions improve performance (Braun et al., 2026). Mutual fund firms choose which funds each manager runs, and these decisions create value (Berk et al., 2017).

As managers have diminishing returns and are homogeneous, the optimal assignment splits capital across them as evenly as possible. Therefore, at every instant, a firm (i, m) produces $m\tilde{f}(i/m)$.[6] Production exhibits constant returns to scale, as doubling both i and m exactly doubles output in the firm. In this way, production does not favor large firms.

The third technology is search in the external market. Outside a firm, placing investor capital with a manager requires search. I think of this search as arising because it takes time to find, evaluate, and contract in the external market. These frictions may stand in for information issues arising from unmodeled heterogeneities (e.g., managers' skill and investors' willingness to commit capital) and for contracting problems. I think of the firm as resolving these issues for each new recruit when he joins. A match pairs one searching investor with one searching manager. Matches arrive as independent Poisson events, and a matching function $\chi(\cdot, \cdot)$ regulates their flow. When a mass n_M of managers and a mass n_I of investors search for each other, matches arrive at flow $\chi(n_M, n_I)$. I assume χ has constant returns to scale.

Define tightness as the ratio of investors to managers: $\theta := n_I/n_M$. Each searching manager then matches at rate $p_M(\theta) = \chi(1, \theta)$ and each searching investor at rate $p_I(\theta) = \chi(\theta^{-1}, 1)$. Note that p_M^{-1} is the tightness required to deliver a given manager-side match rate. Likewise, $1/p_I^{-1}$ is the reciprocal of the tightness required to deliver a given investor-side match rate. I assume χ is such that these two functions are strictly increasing and strictly convex, with value and derivative zero at zero. These assumptions imply that if matching has even a small benefit, some mass of investors and managers will choose to search for each other, as formalized later. For example, a Cobb–Douglas matching function satisfies all of these assumptions.

Search is always between investors and managers. For brevity, from the next subsection on, I say that the firm meets new investors and managers, but I specifically mean that its managers meet unmatched investors and its investors meet unmatched managers. A recruited investor whom one of the firm's managers meets joins the whole firm, as if every manager of the firm had met him. The same holds for a recruited manager whom

[6] Any firm output with constant returns to scale has the form $m\tilde{f}(i/m)$. Let $F(i, m)$ be the output of a firm (i, m), defined at whole numbers, with $F(ki, km) = kF(i, m)$ for every whole number $k \geq 1$. If $i/m = i'/m'$, then $m'F(i, m) = F(im', mm') = F(i'm, m'm) = mF(i', m')$. Hence $\tilde{f}(i/m) := F(i, m)/m$ is well defined at every non-negative rational. If F is also weakly increasing and has diminishing returns to capital for a fixed number of managers, $\tilde{f}$ is weakly increasing and concave, and it has a unique continuous extension to the positive reals. A firm with no managers produces nothing, so I set $m\tilde{f}(i/m) = 0$ at $m = 0$.

one of its investors meets. Suppose θ_I and θ_M are the tightnesses in the firm's searches for investors and for managers. Then, a firm (i, m) meets new investors at rate $m\, p_M(\theta_I)$ and new managers at rate $i\, p_I(\theta_M)$. I model search this way so that the firm's boundary does not change the matching technology.

2.3 The Search Problem

I model the search problem via the market utility approach (see the survey by Wright et al., 2021). One side posts offers that specify a transfer and a tightness. The other side directs its search to the best offer. Firms post offers to unmatched investors and to unmatched managers.[7] The firm can recruit faster by offering larger payments.

Consider first the recruiting of unmatched investors. An offer by a firm of size (i, m) is a pair (R, θ_I). The payment R is the present value of transfers promised to the new investor if the match occurs. Tightness θ_I is the mass of unmatched investors who search for this offer per manager in the firm. The firm meets new investors at Poisson rate $m\, p_M(\theta_I)$. An unmatched investor who searches for this offer matches at rate $p_I(\theta_I)$.

In equilibrium, the tightness that the firms post must be consistent with unmatched investors' choices. Let U_I, the market utility, denote the equilibrium value that an unmatched investor attains by choosing to search for the best posted offer. Because unmatched investors choose the best offer, every offer that attracts any investors in equilibrium must deliver the market utility to unmatched investors:

$$(r + \epsilon_I)U_I = p_I(\theta_I)[R - U_I]. \qquad \text{(Indifference Condition-I)}$$

The left-hand side is the flow value that, in equilibrium, an unmatched investor must earn while he searches for a specific firm's offer. The right-hand side is the rate at which he matches, $p_I(\theta_I)$, times his gain from the match, the promised payment net of the option from continued search he gives up. An especially large payment attracts more unmatched investors. The crowd of investors lowers the rate at which any one of them matches through the offer. Therefore, in equilibrium, searching for the largest payment is no better than searching for any other offer that also attracts investors.

Equation (Indifference Condition-I) pins down the payment a firm must promise for

[7] Which side makes the offers turns out not to matter for match rates or for the division of surplus. I also use this indifference to who posts in Lemma 2 in the proof of Proposition 1, a main result of this paper, which shows that dynamic considerations make firm value exhibit increasing returns.

a given tightness it posts:

$$R = U_I + (r + \epsilon_I)U_I/p_I(\theta_I).$$

The first term means that, regardless of the match rate, the firm must offer at least U_I. Even an unmatched investor who meets the firm instantly will not join unless joining is at least as valuable as continued search. The second term compensates unmatched investors so that the firm matches faster.[8]

Offers to unmatched managers follow the same logic. The firm posts an offer (W, θ_M), unmatched managers search for the best offer, and U_M denotes the unmatched managers' market utility. Tightness θ_M is the number of the firm's investors per unmatched manager who searches for the offer. Because tightness is the ratio of investors to managers, faster recruiting on this side of the search requires a lower θ_M. A new manager receives $W = U_M + (r + \epsilon_M)U_M/p_M(\theta_M)$.

Firms in the same state (i, m) post the same offer on each side. The assumptions on χ make recruiting costs strictly convex and the optimal offer in each state unique.[9]

2.4 Firm's Problem

Recall that a firm consists of investors and managers connected by an internal market. Let $v(i, m)$ denote the joint surplus from keeping a firm (i, m) together. It is the expected present value of the firm's output in excess of the market utilities its current members would obtain by searching without the firm and of the payments R and W to future recruits.

The surplus $v(i, m)$ can also be thought of as the value of the residual claim of the firm's founder, the manager who started the firm. Because agents have a common discount rate, have transferable utility, and can commit to transfers, how the firm pays a member does not matter as long as it delivers the promised present value. I think of the founder as paying each member his market utility as a flow and each recruit the remaining part of his promised payment, $R - U_I$ or $W - U_M$, up front when he joins.[10] The founder

[8]Specifically, to achieve faster matching, the firm needs greater tightness. Greater tightness disadvantages each unmatched investor because more investors compete for the same offers and each waits longer to match. The second term equals the tightness θ_I times each unmatched investor's forgone flow value $(r + \epsilon_I)U_I$, divided by $p_M(\theta_I)$.

[9]Identical postings and the unmatched agents who search for them pool into a search submarket. This pooling closes the microfoundation of search. The matching function χ applies to the masses searching in each submarket and delivers the match rates above.

[10]Payments stop when a member exits, so he effectively discounts them at rate $r + \epsilon_I$ or $r + \epsilon_M$. A flow

keeps the output. His claim is then worth exactly $v(i,m)$. I therefore call $v(i,m)$ firm value. His optimal decisions are the ones the members would make to maximize their joint surplus. The founder still owns the firm even after he exits the labor market.

To make later exposition about the firm's growth easier, I describe the firm's posted offers as specifying its rates of meeting new investors per manager and new managers per investor, $\mu_I := p_M(\theta_I)$ and $\mu_M := p_I(\theta_M)$, which imply the tightness and the payment. The upfront payment is the cost to the firm:

$$\bar{R}(\mu_I; U_I) := R - U_I = (r+\epsilon_I)U_I / p_I(p_M^{-1}(\mu_I)), \qquad \text{(Cost to Firm-I)}$$

$$\bar{W}(\mu_M; U_M) := W - U_M = (r+\epsilon_M)U_M / p_M(p_I^{-1}(\mu_M)). \qquad \text{(Cost to Firm-M)}$$

The prior assumptions on the matching function imply that firms can choose their recruiting rate, and each recruit costs the firm an amount that increases in the rate. The expected cost per unit of time, $\mu\bar{R}(\mu; U_I)$ per manager or $\mu\bar{W}(\mu; U_M)$ per investor, is strictly convex in μ.[11]

The firm chooses its recruiting rates to maximize $v(i,m)$:

$$\begin{aligned} r\,v(i,m) = \max_{\mu_I \geq 0,\, \mu_M \geq 0} \; & m\tilde{f}(i/m) - i(r+\epsilon_I)U_I - m(r+\epsilon_M)U_M \\ & + m\,\mu_I\,[v(i+1,m) - v(i,m) - \bar{R}(\mu_I; U_I)] \\ & + i\,\mu_M\,[v(i,m+1) - v(i,m) - \bar{W}(\mu_M; U_M)] \\ & + i\,\epsilon_I\,[v(i-1,m) - v(i,m)] \\ & + m\,\epsilon_M\,[v(i,m-1) - v(i,m)]. \end{aligned} \qquad \text{(Firm's HJB)}$$

The firm exits at $(0,0)$ and has value $v(0,0) = 0$.

The first line is flow production net of flow payments to existing members. The firm assigns its current investors' capital to its current managers at no cost, and $m\tilde{f}(i/m)$ is the output of that assignment. The second and third lines are changes in firm value, net of the cost to the firm, when the firm matches with unmatched investors and managers in the external market. The last two lines are changes in firm value when members depart.

Recruiting a new member takes time and costs the firm. While searching to repair opposite imbalances, firms forgo output they could have attained if they were balanced. Search frictions therefore matter through both recruitment costs and the delay in realizing

of $(r+\epsilon_I)U_I$ to an investor, or $(r+\epsilon_M)U_M$ to a manager, is therefore worth exactly his market utility.

[11] Note that the costs are zero at a zero rate: $\bar{R}(0; U_I) = \bar{W}(0; U_M) = 0$. These properties follow from the assumptions on p_M^{-1} and $1/p_I^{-1}$, since $\mu\bar{R}(\mu; U_I) = (r+\epsilon_I)U_I\, p_M^{-1}(\mu)$ and $\mu\bar{W}(\mu; U_M) = (r+\epsilon_M)U_M / p_I^{-1}(\mu)$.

these gains in output. However, the firm does not wait or pay again when it later moves capital or managers in its internal market.

Departures of members may push firms to become investor-heavy or manager-heavy. The recruiting rates μ_I and μ_M optimally close these imbalances through the external market. Because $\tilde{f}$ is concave, recruiting a new manager raises production more when each manager already manages a lot of capital, and recruiting a new investor raises production more when each manager manages little. Therefore, investor-heavy firms have an incentive to recruit managers faster, while manager-heavy firms have an incentive to recruit investors faster. Moreover, the greater the gain from adjusting the investor-to-manager ratio, the faster firms adjust over time. However, faster recruiting raises the cost per recruit. Therefore, firms prefer to adjust gradually over time rather than all at once. Later estimation infers how costly search is partly from the speed at which firms close gaps in their investor-to-manager ratios.

For the firm, search acts as a convex cost of adjusting its numbers of investors and managers. The magnitude of this cost is set in equilibrium.

2.5 Entry Problem

Entry is the direct manager-investor version of the same search problem described from the firm's perspective earlier. Rather than search for an incumbent firm's offer, an unmatched manager can instead post his own offer to unmatched investors. Unmatched investors can in turn search for this offer rather than an incumbent firm's offer. A successful match creates a new firm at $(i = 1, m = 1)$.

The manager's offer faces the same investor search problem as an incumbent firm's offer. Let μ_E denote the rate at which the manager recruits his first investor. Because the offer must deliver investors their market utility, inducing a recruiting rate μ_E costs the manager $\bar{R}(\mu_E; U_I)$ upon matching. Upon a successful match, the manager becomes the founder, owns the firm's residual claim, $v(1,1)$, and works for the firm. He is therefore compensated for his labor at U_M as a flow payment as long as he works. U_M is his market utility from pursuing incumbent offers. In this case, the founder receives, in present value, $U_M + v(1,1) - \bar{R}(\mu_E; U_I)$, so the net gain over continuing to search is $v(1,1) - \bar{R}(\mu_E; U_I)$. This $v(1,1)$ includes the possibility that the firm grows large and the founder continues receiving surplus from future production even when he no longer works as a manager for the firm.

Recall that the manager can search for the best incumbent offer and attain the man-

ager market utility U_M. Therefore, in an equilibrium with positive entry, the manager must be indifferent between entry and searching for an incumbent offer:

$$(r+\epsilon_M)U_M = \max_{\mu_E \geq 0} \left\{ \mu_E [v(1,1) - \bar{R}(\mu_E; U_I)] \right\}. \qquad \text{(Entry Indifference)}$$

Here μ_E is the choice variable inside the entry maximization, while μ_E^* denotes its equilibrium optimizer.[12]

Whenever μ_E^* is positive, the first-order condition of the entry maximization and equation (Entry Indifference) imply

$$\begin{aligned} v(1,1) &= U_I \frac{r+\epsilon_I}{p_M'(p_M^{-1}(\mu_E^*))} \\ &= U_M \frac{r+\epsilon_M}{\mu_E^* - p_M^{-1}(\mu_E^*) p_M'(p_M^{-1}(\mu_E^*))}. \end{aligned} \qquad \text{(Relative Market Utilities)}$$

Thus, μ_E^* determines both market utilities relative to the value of a newly founded firm, $v(1,1)$.

2.6 Equilibrium

To close the economy, I now account for the fixed populations of investors and managers. I focus on a stationary equilibrium with positive entry. Let $g(i,m)$ denote the mass of firms at state (i,m) and let γ_E denote the flow of new firms. Firms at the same state use the same optimal recruiting rates $\mu_I^*(i,m)$ and $\mu_M^*(i,m)$.

The investor population I is allocated across three parts: investors already inside incumbent firms, unmatched investors searching for incumbent offers, and unmatched investors searching for entry offers. These three parts must sum to I. This is the aggregate investor market-clearing condition.

Consider firms at state (i,m). Each firm already has i investors, so a mass $i\,g(i,m)$ of investors is already allocated to these firms. Each firm recruits new investors at total rate $m\,\mu_I^*(i,m)$.

Recall that the tightness of a firm's offer is the mass of unmatched investors who search for the offer per manager in the firm. Therefore, a recruiting rate $\mu_I^*(i,m)$ requires tightness $p_M^{-1}(\mu_I^*(i,m))$. Since each firm has m managers, a mass $m\,p_M^{-1}(\mu_I^*(i,m))$ of

[12] A firm with one manager and no investors faces the same problem as an unmatched manager posting an entry offer. Lemma 2 shows that such a firm has no firm value, $v(0,1) = 0$. The manager's market utility U_M already includes all future value from that state. Setting $v(0,1) = 0$ and $v(0,0) = 0$ in equation (Firm's HJB) at $(i,m) = (0,1)$ makes its left-hand side zero, and the remaining terms give equation (Entry Indifference).

unmatched investors must be searching for its offer. Thus, the total mass of unmatched investors searching for offers of firms at this state is $m\, p_M^{-1}(\mu_I^*(i,m))\, g(i,m)$. Summing the investors already inside firms and those searching for incumbent offers over all firm states gives the investor population allocated to incumbents.

Recall that an unmatched manager posting an entry offer matches at the equilibrium rate μ_E^*, and each successful match creates one new firm. Therefore, the flow of entrants is

$$\gamma_E = \mu_E^* \times \{\text{mass of unmatched managers posting entry offers}\}.$$

The mass of unmatched managers posting entry offers is therefore γ_E/μ_E^*. The tightness associated with μ_E^* is $p_M^{-1}(\mu_E^*)$. Since tightness is the ratio of investors to managers, the mass of unmatched investors searching for entry offers is $(\gamma_E/\mu_E^*)p_M^{-1}(\mu_E^*)$. Putting everything together, aggregate investor market clearing requires

$$I = \sum_{(i,m)} g(i,m)\left[i + m\, p_M^{-1}(\mu_I^*(i,m))\right] + \frac{\gamma_E}{\mu_E^*} p_M^{-1}(\mu_E^*). \qquad \text{(Aggregate Investor Market Clearing)}$$

The manager population consists of managers already inside incumbent firms, unmatched managers searching for incumbent offers, and unmatched managers posting entry offers. Aggregate manager market clearing is analogous:

$$1 = \sum_{(i,m)} g(i,m)\left[m + \frac{i}{p_I^{-1}(\mu_M^*(i,m))}\right] + \frac{\gamma_E}{\mu_E^*}. \qquad \text{(Aggregate Manager Market Clearing)}$$

Next, I define a stationary equilibrium in which agents optimize match rates and markets clear.

Definition 1. *A stationary equilibrium is a collection*

$$\{U_I, U_M, v(), g(), \mu_I^*(), \mu_M^*(), \mu_E^*, \gamma_E\}$$

such that the following hold.

First, unmatched investors and managers choose optimally among the active offers described in Sections 2.3 and 2.5. Every active offer delivers the relevant market utility, and the masses choosing each offer imply the tightness associated with its recruiting rate.

Second, given U_I and U_M, $v()$ solves the HJB in equation (Firm's HJB), and $\mu_I^*(i,m)$ and $\mu_M^*(i,m)$ are the optimal recruiting policies.

Third, unmatched managers solve the entry problem in equation (Entry Indifference), and μ_E^* is the optimal rate at which a manager who posts an entry offer recruits his first

investor.

Fourth, the mass of firms at each state, $g()$, is stationary under the optimal recruiting policies, member departures, and the entry flow γ_E, and both aggregate market-clearing conditions hold. Stationarity means that inflows equal outflows at every firm state.

Firms enter state $(1,1)$ at flow rate γ_E. A firm at (i,m) recruits investors at total rate $m\mu_I^*(i,m)$ and managers at total rate $i\mu_M^*(i,m)$, while investors and managers in the firm exit at total rates $i\epsilon_I$ and $m\epsilon_M$. Appendix A.4 writes out the resulting laws of motion.

I compute the equilibrium with the following algorithm. Given μ_E^*, the entry conditions determine U_I and U_M relative to $v(1,1)$. The HJB, with U_I and U_M at these multiples of $v(1,1)$, then determines the optimal incumbent recruiting policies. Given these policies, the laws of motion are linear in $g()$ and γ_E. Thus, multiplying the entry flow by a constant multiplies the mass of firms at every state by the same constant without changing the relative firm-size distribution. Aggregate manager market clearing pins down this scale by determining γ_E, and therefore the magnitude of $g()$. Given the resulting γ_E and $g()$, the equilibrium μ_E^* is the value for which aggregate investor market clearing holds.

3 Theoretical Results

This section presents the paper's three main theoretical results, emphasizing the dynamic considerations that make relationships valuable.

Recall that output and search scale proportionally with size. However, Proposition 1 says that firm value exhibits increasing returns to relationships due to dynamic considerations.

Proposition 1. Strictly Super-Additive Firm Value: *Under the maintained assumptions of the model, and assuming firm value is finite,*

$$v(i+i', m+m') > v(i,m) + v(i',m')$$

for all $(i,m),(i',m') \in \mathbb{Z}_+^2 \setminus \{(0,0)\}$. Firm value is therefore strictly super-additive in its numbers of investors and managers. Splitting a firm into two strictly lowers total firm value.

Proof. Appendix B.1. □

Investors and managers depart over time, and the side that remains then needs a new counterpart. In a large firm, the remaining side is instead reassigned to the firm's other members without new search, and these future options to reassign are the dynamic considerations behind Proposition 1. Specifically, combining two identical firms adds no output today but can reduce the need for external search after future departures.

Investors and managers depart independently. Therefore, departures that in many cases barely move the combined firm's ratio can instead leave one of the two firms investor-heavy and the other manager-heavy. The combined firm's advantage is a steadier balance between investors and managers, not necessarily a steadier total size.[13] Because $\tilde{f}$ is concave, a steadier ratio of investors to managers increases output across time. Apart, imbalanced firms recruit in the external market to repair their imbalances, incurring recruitment costs and forgoing this additional output while search continues.

However, a combined firm moves managers, without search, away from the part that would have been manager-heavy to the part that would have been investor-heavy had the two firms been separate. As a result, the combined firm runs less into diminishing returns to capital per manager than the two separate firms do. Therefore, until the two separate firms repair their imbalances, the combined firm can produce more than they do. Moreover, the combined firm may not need to pay search costs as high as the two separate firms do because it has less urgency to repair imbalances. The logic of Proposition 1 has implications for how the investor-to-manager ratios of small and large firms move. These implications hold in the untargeted moments of Section 4.7.

The boundary of a firm depends on how costly relationships are to form in the external market and how valuable they are to use in its internal market. Proposition 1 implies that a firm would expand instantly if expansion were free. However, expanding requires search, which is also why a large firm is valuable in the first place. I quantify this tradeoff in Figure 3 of Section 5. In principle, two firms could also capture the gain in Proposition 1 by merging. However, it is unclear whether a merger would be free of the problems I model as search. A merger brings many investors and managers into one firm at once. The merged firm still has to evaluate and contract with each of them before it can reassign them freely, which may take time and be costly. If a merger could be done

[13] Diamond (1984) shares the logic that combining independent shocks inside one larger firm can be valuable. There, an intermediary that lends to many borrowers with independent returns rarely defaults. Here, the shocks are departures that unbalance investors and managers in production. In a large firm, departures on the two sides offset each other. Because restoring the balance between the two sides requires search, the value of combining is dynamic, and it shapes firm dynamics and the equilibrium.

instantly, search frictions would no longer constrain how the economy allocates capital and labor.

Corollary 1 shows that even after compensating a new manager, the founder captures value from this expansion. One way to look at the cost of expansion to the firm relative to the value it creates is a labor share. Offers in the search problem specify the present value of payments to a recruit. Therefore, I define the labor share of a marginal manager in present value.

Definition 2. Ex-Ante Marginal Labor Share: *Consider a firm at state* (i, m) *with* $i \geq 1$ *that subsequently adds one manager. In present value when he joins, before he starts working, the manager receives his market utility plus the upfront payment,* $U_M + \bar{W}(\mu_M^*(i, m); U_M)$*. The firm receives* $v(i, m+1) - v(i, m) - \bar{W}(\mu_M^*(i, m); U_M)$*. These values sum to* $v(i, m+1) - v(i, m) + U_M$*. Define the manager's share of this combined value as*

$$l(i, m) := \frac{U_M + \bar{W}(\mu_M^*(i, m); U_M)}{v(i, m+1) - v(i, m) + U_M}.$$

This definition is closer to a conventional labor share than to a division of surplus. The manager would get market utility U_M by continued search outside the firm. Therefore, the firm's payment of U_M is not a transfer of surplus to the manager. The definition nonetheless includes this market utility because conventional labor shares are not net of opportunity costs. Corollary 1 says that this share is below one for any firm with at least one investor.

Corollary 1. Labor Share Below One: *Under the maintained assumptions,*

$$l(i, m) < 1$$

for every (i, m) *with* $i \geq 1$*. A new manager therefore receives less than the full marginal value from expanding the firm, and the firm keeps the rest (i.e., the founder receives the rest).*

Proof. Appendix B.2. □

Search frictions have two consequences. First, as Proposition 1 shows, a firm is worth more than the sum of its parts because its members avoid future search frictions when they need to be reassigned. Second, as Corollary 1 shows, the firm can grow larger without having to pay the entire marginal value because search frictions blunt competition among firms for managers.

Each firm has to make greater payments to induce a desired match rate when other firms are trying to match quickly. This competition is what in equilibrium sets the manager market utility. A firm that pays slightly more than others will not attract all unmatched managers because unmatched managers crowd each other out and lower their value from searching for the firm. Therefore, search frictions make firms compete less with each other than they would without frictions.

The last main result is about the frictionless limit of the model. Definition 3 introduces a uniform scaling for the matching function.

Definition 3. Matching Efficiency: *Take the current primitives and consider an economy in which the matching function is scaled by a matching efficiency $\lambda > 0$:*

$$\chi_\lambda(n_M, n_I) := \lambda\, \chi(n_M, n_I). \qquad \text{(Scaled Matching Function)}$$

The corresponding match rates are

$$p_{M,\lambda}(\theta) = \lambda\, p_M(\theta), \qquad p_{I,\lambda}(\theta) = \lambda\, p_I(\theta).$$

Thus, a larger λ uniformly speeds external matching at every tightness. All other primitives are held fixed. Let $U_{I,\lambda}$ and $U_{M,\lambda}$ denote the equilibrium market utilities and $v_\lambda(i,m)$ denote firm value in the economy with λ scaling. The frictionless limit is $\lambda \to \infty$.

When matching becomes infinitely fast, Proposition 2 says that firm value at each fixed state and the value created by combining each fixed pair of firms converge to zero.

Proposition 2. Frictionless Limit: *Under the maintained assumptions and assuming an equilibrium exists for all sufficiently large λ,*

$$v_\lambda(i,m) = O(\lambda^{-1})$$

for every fixed $(i,m) \in \mathbb{Z}_+^2$. Thus, firm value vanishes pointwise: $\lim_{\lambda\to\infty} v_\lambda(i,m) = 0$. For every fixed pair of nonempty states, the value created by combining the two firms also vanishes:

$$v_\lambda(i+i', m+m') - v_\lambda(i,m) - v_\lambda(i',m') \longrightarrow 0.$$

Proof. See Appendix B.3. □

Market utilities are bounded by total output and thus do not explode as λ increases. As λ increases, a firm and the unmatched agents searching for its offers match increasingly fast. The cost to the firm for any given match rate is the market utility times a

term that collapses as λ increases. This cost therefore vanishes. In the limit, the firm matches instantly without having to pay any amount above market utility to induce more unmatched agents to search for its offers. A new recruit cannot raise the firm's value because current value already prices in this match. A firm worth zero today is therefore still worth zero after one more recruit.

Recall that an entrant is an unmatched manager who recruits an investor. The manager starts with no firm value and only has his market utility, $U_{M,\lambda}$. Once he recruits his first investor and creates a new firm, he gains firm value $v_\lambda(1,1)$. However, $U_{M,\lambda}$ already prices the option of this entry. With frictionless search, entry comes instantly and free. In the limit, an unmatched manager's value $U_{M,\lambda}$ therefore equals the value of entering at once, $U_{M,\lambda} + v_\lambda(1,1)$. Hence $v_\lambda(1,1) \to 0$. Every other firm grows from $(1,1)$ one recruit at a time, and by the previous paragraph each recruit keeps the firm's value at zero.

When search becomes easy for everyone, an investor-heavy firm and a manager-heavy firm can adjust their investor-to-manager ratios instantly through the external market. Existing relationships then provide no advantage in avoiding future external search. Therefore, a combined firm has no advantage over two separate firms.

The baseline economy is built so that only relationships may generate firm value. I next extend the model so that firms may also own other intangibles. Then I estimate the model and quantify how much of firm value comes from reduced dependence on future external search.

4 Quantification

4.1 Adding Other Intangibles

I introduce a third state variable, $Z > 0$, to give firms a reason to be valuable beyond reusing relationships. This Z represents all other intangibles the firm owns aside from its stock of relationships, which itself could be seen as an intangible.

Production is $Z^{1-\nu}\big(m\tilde{f}(i/m)\big)^{\nu}$, where $\tilde{f}(k) = k^{1-\alpha}$ at whole numbers k and $\tilde{f}$ is linear between adjacent whole numbers, with $\alpha \in (0,1)$ and $\nu \in (0,1)$.[14] Wherever i/m

[14]With this $\tilde{f}$, $m\tilde{f}(i/m)$ is the firm output $F(i,m)$ of Section 2.2 when each investor's unit of capital goes to one manager and a manager with k units produces $k^{1-\alpha}$. That is, $F(i,m) = \max_{\sum_{j=1}^m k_j = i} \sum_{j=1}^m k_j^{1-\alpha}$ over $k_j \in \mathbb{Z}_+$. Write $i = qm + \ell$ with $q, \ell \in \mathbb{Z}_+$ and $\ell < m$. As $k^{1-\alpha}$ is concave, the most even split is optimal. It gives ℓ managers $q+1$ units and the other $m-\ell$ managers q units. Its output $(m-\ell)q^{1-\alpha} + \ell(q+1)^{1-\alpha}$ equals $m\tilde{f}(i/m)$ because $i/m = q + \ell/m$ and $\tilde{f}$ is linear between q and $q+1$.

is a whole number, production equals $Z^{1-\nu}\left(i^{1-\alpha}m^{\alpha}\right)^{\nu}$, so α is the weight on managers and $1-\nu$ the weight on intangibles.

Output has constant returns in (Z, i, m). A firm with twice the intangibles, investors, and managers produces twice as much, so production is neutral to firm size, as in the baseline model. Holding Z fixed, output instead has decreasing returns in (i, m), with degree ν.

I think of $z := Z^{1-\nu}$ as a productivity process that follows a random walk in logs. Specifically, $\log(Z)$ follows, independently across firms,

$$d\log Z = \left(\mu_Z - \tfrac{\sigma_Z^2}{2}\right)dt + \sigma_Z\, dB, \qquad \mu_Z := (1-\zeta)\tfrac{\sigma_Z^2}{2}$$

where B is a Brownian motion and σ_Z controls the volatility of the process. With $\zeta > 1$, μ_Z is negative and Z declines on average. The right tail of Z across firms is then Pareto with index ζ.[15]

New firms enter at $(i = 1, m = 1)$ with $\log(Z)$ drawn from a normal distribution with mean $-s_Z^2/2$ and standard deviation s_Z. This mean sets the average Z at entry to one, which normalizes the units of Z. I use this distribution to describe entry dispersion with one parameter. Incumbents with a high z have an advantage in producing due to their intangibles. Intangibles may therefore create additional value for firm owners besides the reuse of relationships I highlighted in the baseline model.

I assume the matching function is Cobb–Douglas with an elasticity of one half: $\chi(n_M, n_I) := \lambda\sqrt{n_M\, n_I}$, where the matching efficiency $\lambda > 0$ uniformly scales matching. Then $p_M(\theta) = \lambda\sqrt{\theta}$ and $p_I(\theta) = \lambda/\sqrt{\theta}$. With this matching function, the costs to the firm in (Cost to Firm-I) and (Cost to Firm-M) are linear in the match rate and inversely proportional to λ^2.

Given the heterogeneity and randomness of z, some firms may prefer to release managers or investors. The baseline model does not need this option. In this extension, I therefore allow for free disposal of members. A firm can release any member at no cost, and the released managers and investors become unmatched and return to the external market. Appendix D.1 states the firm's HJB.

4.2 Data

The data come from annual Form ADV filings by investment advisers registered with the Securities and Exchange Commission (SEC). I use *adviser* to refer to the entity

[15] I require $\zeta > 1$ so that mean firm size is finite.

submitting a filing. An asset management firm can operate through several advisers.

Each filing includes information about the money the adviser manages and the number of employees it employs. The main variable is regulatory assets under management (RAUM). RAUM is the value of portfolios to which the adviser provides ongoing supervisory or management services as defined by regulations. It is gross of liabilities and includes uncalled commitments in private funds.

Filings also distinguish how much money the adviser manages through pooled vehicles and how much through separately managed accounts (SMAs) for each client. The pooled vehicles include mutual funds, hedge funds, and private equity funds. For SMAs, the form also reports the types of assets held, including holdings in pooled vehicles.

The filings report how many employees perform investment advisory functions, including research. I refer to these employees as advisory staff. The filings also report links to other advisers through ownership and control, and whether the advisers share supervised persons.[16]

I think of the boundary of the firm as the labor or capital the firm can reassign. There are legal and tax reasons for a firm to file through separate advisers. But I view separate advisers as part of the same economic firm, and thus I consolidate them. A parent can reassign staff and capital across its advisers. I consolidate two advisers when each reports the other as related through control and as sharing supervised persons.

Advisers in one firm may report some of the same staff and assets if both advisers are considered to use the same staff or service the same capital. Appendices C.2 and C.3 describe how I consolidate advisers and remove the double counting.

I map the model's investor base, i, to the capital that the firm's managers actively invest, and its manager base, m, to advisory staff. RAUM includes funds that are managed passively, following an index. Therefore, they are not part of i. I link registered funds (e.g., mutual funds) to annual Form N-CEN reports and the CRSP Mutual Fund Database. I remove passive funds from RAUM.

I further adjust RAUM by removing SMA holdings that firms report having put in pooled vehicles. The adviser of each pooled vehicle, in this firm or another, already reports these assets. Because filings do not report whether remaining SMAs are passive, I use the active share of the firm's funds as a proxy for the active share of its remaining SMA assets. I subtract the passive SMA assets from RAUM. I call the resulting measure

[16]Supervised persons are the officers, employees, and others who give investment advice on an adviser's behalf under its supervision. The reported fields are in Form ADV Part 1A, Items 5.B, 5.D, and 5.F, and Schedule D, Sections 5.K and 7.A.

active assets under management (active AUM). For brevity, I write AUM for active AUM from here on.

I keep only firm-years with at least $90M of RAUM. Below it, advisers keep filing only under exceptions. I drop firms based outside the U.S. They register only after their U.S. business outgrows an exemption. I drop firms with a third or more of their active AUM in SMAs. An SMA follows one client's own mandate, not the firm's assignment of capital to its managers. The cut removes mainly wealth managers, the asset managers of insurers, and managers of large institutional accounts. Appendix C.5 provides the details.

I identify entrant firms by looking at their earliest SEC filing. Advisers generally must register with the SEC once their RAUM reaches $110M (Appendix C.1). Therefore, an entrant is a firm that has just become large enough for regulators to require it to report. It is still small relative to many incumbents. The median entrant has $208M of active AUM and 4.6 advisory employees, against $830M and 8 for the median firm-year (Table 1).[17] Some firms first file only when they are already large, for example a new adviser of an established group or a foreign manager entering the U.S. I check firms with first filings that report more than $500M of active AUM against public records and correct their entry dates if needed. For new advisers formed by existing firms, I use the existing firm's founding year (Appendix C.4). I map $(i, m) = (1, 1)$, the entry size in the model, to the median entrant.

The sample covers 2018 to 2025 and contains 5,485 firms and 27,736 firm-years, including 1,463 entrants. Table 1 reports the sample counts, the distribution of firm size, and annual growth.

To measure the share of value added paid to labor, I obtain compensation, revenue, and non-labor expenses from the annual segment reports of the public parents of firms in the sample from 2022 to 2025 (Appendix C.6).

4.3 Moments

Time is continuous in the model, while the data are annual. I therefore build an annual panel from the model. I simulate firms from entry year by year, recording i and m at the end of each year. In the data and in the model, I winsorize the levels and annual changes of $\log(i)$ and $\log(m)$ at the 2.5th and 97.5th percentiles within each year to reduce

[17] Staff counts are integers, and nearly half of entrants have four or fewer advisory employees. Therefore, the ordinary median jumps between 4 and 5 across the bootstrap draws of Section 4.4. I instead treat each count k as spread evenly over $[k - \frac{1}{2}, k + \frac{1}{2}]$ and take the median of that distribution.

Table 1: Summary Statistics

Panel A. Firm-years

	Mean	S.d.	P10	P25	Median	P75	P90
Size							
Active AUM ($B)	10.74	101.20	0.17	0.31	0.83	2.82	10.78
Advisory staff	30.2	246.8	2.0	4.0	8.0	17.0	41.0
AUM per advisory employee ($M)	257	752	31	57	113	237	497
Annual growth $(100\,\Delta \log)$							
Active AUM	9.0	40.6	−27.3	−7.7	6.7	24.6	49.6
Advisory staff	2.7	26.6	−19.4	0.0	0.0	11.1	28.8

Panel B. Sample

	Count
Adviser filings, one per adviser and year	32,170
Advisers	6,027
Firms	5,485
Firm-years	27,736
Entrant firm-years	1,463
Observations in annual growth statistics	22,493
Firm-years in growth regressions	13,846

Panel C. Entrants

	Median
Active AUM ($M)	208
Advisory staff	4.6

The table pools firm-years from 2018 to 2025 with equal weight. Size and growth use constructed active AUM and advisory staff before statistical adjustments for extreme observations, business mix, or years (Section 4.3). Growth uses consecutive years, combining merging firms in both years so that a merger does not appear as growth (Appendix C.2).

the influence of outliers (Appendix C.7). Section 4.5 later explains why these moments identify the parameters.

Dynamic Moments. The baseline theory emphasizes shocks that disturb $\log(i/m)$ and how firms choose μ_I, μ_M to optimally undo those disturbances through the external market. I therefore examine how (i, m) evolves over time and how current states predict it. I run the following two regressions for the change in year 1 and in year 2 ($h \in \{1, 2\}$):

$$\Delta \log(i_{t+h}) = \beta_h^I \log(i_t/m_t) + \rho_h^I \Delta \log(i_t/m_t) + \delta_h^I X_t + e_{t+h}^I$$

and

$$\Delta \log(m_{t+h}) = \beta_h^M \log(i_t/m_t) + \rho_h^M \Delta \log(i_t/m_t) + \delta_h^M X_t + e_{t+h}^M,$$

where X_t controls for the firm's current scale and the past change in its scale.[18] I refer to β_h^I and β_h^M, the coefficients of the change in $\log(i)$ and $\log(m)$ on $\log(i_t/m_t)$, as the investor leg and the manager leg. A negative investor leg or a positive manager leg means that the ratio reverts. I refer to $\rho_h^I - \rho_h^M$, the coefficient of the change in $\log(i/m)$ (i.e., combining both regressions) on $\Delta \log(i_t/m_t)$, as the ratio momentum. A negative ratio momentum means that part of last year's change in the ratio reverses. I target these six coefficients, β_h^I, β_h^M, and $\rho_h^I - \rho_h^M$ for $h = 1, 2$. I call the year-2 coefficients increments because they are the coefficients on the change from t to $t+2$ less those on the change from t to $t+1$. Appendix D.2 reports the coefficients on the change from t to $t+2$ as the moments by year 2. In the data and in the model, the regressions use only firms observed from $t-1$ to $t+2$.

Different types of investment products (e.g., a growth equity fund versus a money market fund) may differ in how labor-intensive their production is. Because firms load differently on the investment products they offer, their $\log(i/m)$ may differ because their production functions differ. In the model, all firms have the same production function. Without residualizing this difference, the regressions would interpret lasting differences in ratios due to differences in production functions as disturbances that firms can never undo.

Therefore, I residualize the variables in the regressions on firms' business mix (e.g., the shares of active AUM in mutual funds, hedge funds, and private equity funds, private fund strategies, and client types) observed at t. I also residualize on year indicators (Appendix C.7). Because the business mix is measured at t, every variable on the right-hand side is observable at time t, and the regressions are predictive. Across all firm-years, the residualization removes around 32% of the within-year variance of $\log(i/m)$ (Table 5).

The target moments then compare, for example, firms that are especially investor-heavy relative to other firms with the same business mix, year, and scale.

Size Moments. I target average sizes $\mathbb{E}\log(i)$ and $\mathbb{E}\log(m)$ in the data and in the model's stationary distribution (Appendix C.7). I do not residualize these on business

[18] Specifically, $X_t = \big(\log(i_t\ m_t), \Delta \log(i_t\ m_t)\big)$. I measure scale with $\log(i_t\ m_t)$ because doubling both i and m raises it and leaves $\log(i_t/m_t)$ unchanged.

mix, unlike the dynamic moments. The residuals would have mean zero by construction, and the model should fit the average firm across business mixes. I also target the mean of $\log(i)$ among the top 10% of firms by AUM to fit the top tail of firms.

Dispersion Moments. I also target three dispersion moments. They measure how much firms differ. The first is the standard deviation of $\log(m)$ net of year indicators. The second is the standard deviation of $\Delta \log(m)$, residualized on business mix and measured net of the conditional expectation implied by the dynamic regressions described earlier. The third is the standard deviation of $\log(i/m)$, residualized on business mix and year.

Entry Moments. I target two entry moments. Because investors and managers can also meet directly and start a firm, the model should fit how much entry there is. The first is the entrant share of AUM, the share of all AUM managed by firms that entered in the past year. The second is the entrant share of managers, the same share for managers.

Labor Share. Last, I target the imputed share of corporate value added paid to labor in the top 10% of firms by advisory staff (Appendix C.6). The data measure this flow share, not the marginal present value in Definition 2. Firms with a public parent are a selected sample of the largest firms. I map this moment to the top 10% of firms by m in the model. In the data, I impute labor and value added for firms without a public parent among the 10% of firms with the most advisory staff (Appendix C.6). Firms with a public parent hold 26% of the active AUM of these firms. In the model, this corresponds to manager pay (market utilities and upfront payments) divided by output less investor pay (market utilities and upfront payments).

4.4 GMM

I externally calibrate three parameters as in Table 2.

Table 2: External Calibrations

Parameter	Value	Object
r	0.05	Annual discount rate
ϵ_M	0.18	Manager separation hazard
ϵ_I	0.065	Investor separation hazard

A yearly change in assets or staff is the number of investors or managers who join minus the number who leave. The data therefore do not reveal how many leave, which is what the separation hazards ϵ_I and ϵ_M measure. I calibrate them outside the model, ϵ_M from

the career-transition statistics in Jackson (2025) and ϵ_I from the follow-on reinvestment statistics in Sensoy et al. (2013).[19]

The remaining seven parameters are estimated inside the model.[20] Define

$$\Theta := (\lambda, \log(I), \alpha, 1-\nu, s_Z, \sigma_Z, \zeta)\,.$$

Let $\widehat{\psi}$ denote the vector of targeted data moments. Let $M(\Theta)$ denote the same moments computed in the stationary distribution of the model with the external parameters fixed at the values in Table 2. The moment residual is

$$e(\Theta) = \widehat{\psi} - M(\Theta).$$

Let $\widehat{\Omega}$ be the bootstrap covariance matrix of $\widehat{\psi}$. The estimator solves

$$\widehat{\Theta} = \arg\min_{\Theta \in \mathcal{P}} e(\Theta)' \widehat{\Omega}^{-1} e(\Theta),$$

where $\mathcal{P}$ is the parameter space. I sample whole firm histories in case errors across time are correlated within firms. Standard errors of the estimates use this $\widehat{\Omega}$ in the asymptotic variance of the estimator. Standard errors of model results follow from them by the delta method.

4.5 Identification

Appendix D.2 reports two exercises. The first exercise follows Honoré et al. (2020) to measure how much the asymptotic standard errors of parameter estimates rise when some moments are dropped (see Table 7). The second exercise follows Andrews et al. (2017) and measures how much each estimated parameter would move if a targeted moment were perturbed (see Figure 5). I summarize the results below.

A less negative ratio momentum suggests a larger λ, that is, smaller search frictions. Firms that find investors and managers more easily have less need to spread repairs over multiple years and leave less of last year's change in $\log(i/m)$ to undo. The mean of $\log(i)$ and the standard deviation of $\log(i/m)$ discipline I. Larger values of both suggest a larger mass of investors as firms can accumulate more investors, investors' market utilities are lower due to abundance, and thus firms can tolerate greater variability in their ratios.

A higher labor share suggests a larger α and a smaller $1-\nu$ because managers are better paid when output leans more on them and less on Z. A larger standard deviation

[19] Appendix D.3 explains both values and reports how the estimates and the results change with ϵ_I and ϵ_M.

[20] I report $1-\nu$ because it is the output elasticity of intangibles, which is easier to interpret than ν.

of $\log(m)$ suggests a greater $1-\nu$ because high-Z firms then grow largest and firms differ more in size.

A smaller mean of $\log(m)$ suggests a greater σ_Z because, at a given ζ, $\log(Z)$ then declines faster on average and incumbents are smaller. A larger top-decile mean of $\log(i)$ suggests a larger s_Z because more entrants start with a high Z and grow into the largest firms. Larger entrant shares of AUM and staff suggest a larger ζ because a thinner right tail of Z makes the largest firms smaller relative to entrants. The investor and manager legs and the standard deviation of growth of m discipline several parameters jointly.

4.6 Estimates

Table 3 reports the parameter estimates. I is large, implying investor capital is abundant relative to managers. Heuristically, if all agents were unmatched, it would take around 3.2 years (i.e., $1/(\lambda\sqrt{I})$) for a manager to find an investor and form an entrant, which means raising around $200M and bringing about 5 advisory staff together. The exact entry speed depends on the distribution of incumbent firm sizes (which affects the mass of unmatched investors and managers) and how much the manager chooses to offer to increase his match rate. In equilibrium, a manager who searches to start a firm finds his first investor in 1.6 years on average (i.e., $1/\mu_E^*$).

Production depends more on managers than on investors ($\alpha = 0.872$), and the output elasticity of intangibles is $1-\nu = 0.147$. The volatility σ_Z implies that productivity (or equivalently production at fixed inputs) fluctuates by around 1.7% a year (i.e., $(1-\nu)\sigma_Z$). The entrant distribution of $\log(z)$ has a standard deviation of 55% (i.e., $(1-\nu)s_Z$). Firms exit when they lose all investors and managers, including by releasing them when z is low.

Table 3: Parameter Estimates

No.	Parameter	Estimate	S.E.	Object
1	λ	0.056	0.003	Matching efficiency
2	$\log(I)$	3.434	0.078	Log mass of investors
3	α	0.872	0.038	Exponent in $\tilde{f}(k) = k^{1-\alpha}$
4	$1-\nu$	0.147	0.012	Output elasticity of other intangibles
5	s_Z	3.744	0.072	S.d. of $\log(Z)$ at entry
6	σ_Z	0.115	0.012	Volatility of $\log(Z)$
7	ζ	1.140	0.047	Tail index of Z

Identification: Table 7 and Figure 5 in Appendix D.2.

Table 4 reports the targeted data and model moments. I target 15 moments with 7 parameters. The model generally fits the size and dispersion moments, except the standard deviation of growth of m, and generates the main patterns in the dynamic moments. In the data and in the model, the largest firms are far larger than the average firm, entrants hold very small shares of AUM and staff, firms differ almost as much in $\log(i/m)$ as in $\log(m)$, and labor receives about half of corporate value added at the largest employers. However, entrants' shares are smaller in the model than in the data.

In years 1 and 2, firms move both investor and manager bases to repair an imbalance in $\log(i/m)$, and the investor base moves more than the manager base. A firm's past change in its $\log(i/m)$ is also partly undone. The model generates these patterns, except that in year 1 it moves the two bases about equally. The investor base moves more in year 1 than in year 2, and the model gets this front-loaded shape right. However, it overestimates the extent of the reversion, mainly in year 1. The largest miss is the manager leg in year 1, which is more than twice as large in the model as in the data.

Table 4: Targeted Moments

No.	**Moment**	**Data**	**S.E.**	**Model**	**No.**	**Moment**	**Data**	**S.E.**	**Model**
Size Moments (2)					*Ratio Momentum Moments* (2)				
1	Mean $\log(i)$	1.634	0.028	1.600	10	Year 1	−0.034	0.011	−0.065
2	Mean $\log(m)$	0.661	0.025	0.634	11	Year 2 increment	−0.053	0.012	−0.061
Dispersion Moments (3)					*Investor Leg Moments* (2)				
3	S.d. $\log(m)$	1.098	0.015	1.100	12	Year 1	−0.039	0.004	−0.050
4	S.d. growth of m	0.187	0.003	0.202	13	Year 2 increment	−0.029	0.005	−0.033
5	S.d. $\log(i/m)$	0.891	0.013	0.891	*Manager Leg Moments* (2)				
Entry Moments (2)					14	Year 1	0.022	0.003	0.053
6	Entrants' AUM share	0.002	0.000	0.000	15	Year 2 increment	0.024	0.003	0.028
7	Entrants' staff share	0.010	0.001	0.003					
Labor Share Moments (1)									
8	Top 10% employers	0.537	0.020	0.495					
Top AUM Decile Moments (1)									
9	Mean $\log(i)$	5.522	0.085	5.565					

4.7 Untargeted Moments

Figure 1 reports two sets of untargeted moments at the core of firm dynamics generated by the model.[21] The core idea underlying Proposition 1 is that larger firms need less future search to repair imbalances. This has three implications. Firms with unusually high or low ratios should repair their imbalances over time. These repairs would have been offset inside a larger firm. Departures move a small firm's ratio more than a large firm's, but a larger firm has less incentive to repair imbalances than a smaller firm. I test these ideas by looking at drifts and volatilities of small versus large firms.

First, firms' own choices reveal that they try to repair their imbalances and that the repair takes time. In the first panel, I measure how quickly small firms move their ratios when these ratios are unusually high or low. I take two small firms that are lopsided on opposite sides in year $t-1$. Then, I measure the change in each firm's $\log(i/m)$ from t to $t+2$. The data and model align in the direction of these changes by year 2, but the model repairs faster than the data.

Second, a large firm could have offset these repairs internally and needed less change in its ratio. I compare the sum of the two small lopsided firms with a large firm of their combined size. Because the two repairs go in opposite directions, they cancel in the sum. The sum's ratio then moves like the real large firm's. The two small firms search to close gaps in their ratios that a firm of their combined size does not have. In both the data and the model, the sum's ratio moves little relative to the two repairs.

Third, I show that a small firm's ratio is more volatile than that of a large firm with the same ratio of investors to managers. The second panel takes a large firm and compares it with firms that have about half its i and half its m. I subtract from the change in $\log(i/m)$ its conditional expectation from the regressions in Section 4.3. This removes the predictable repairs of the first panel, leaving future shocks and future repair of those shocks. In the data and the model, small firms have higher variability than large firms with similar ratios of investors to managers.

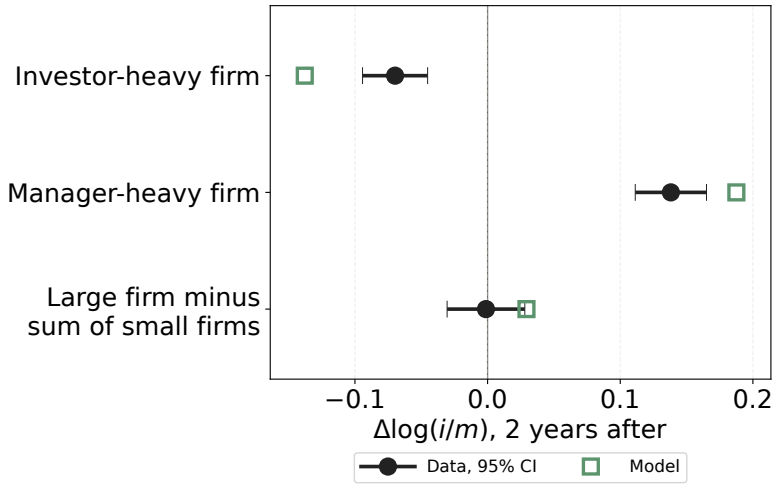
The same panel states this fact in a second way. Each small firm is more imbalanced than the sum of the two and tries harder to repair its imbalances after shocks. A large firm offsets these imbalances internally and does not need its parts to search so intensely. Therefore, the sum of two small firms can be less volatile than a large firm. In the data, I

[21] In both panels, I residualize all firms' i and m states so that the comparisons do not load on differences in business mix (Appendix D.4). I also residualize and winsorize the growth rates as I do with the dynamic moments I target.

sum pairs of small firms in the second panel at each date and compare the sums with real large firms with similar ratios of investors to managers. The sum's ratio is less volatile than the large firm's in both the data and the model. Because two independent firms average their shocks, this comparison carries the same fact as the comparison of small and large firms and not a separate one. In the model, this steadiness is of no use to the two small firms because each produces with its own ratio.

Figure 1: Untargeted: Repairing Imbalances and Ratio Volatility

(a) Repairing imbalances: $\Delta \log(i/m)$



(b) Ratio volatility: s.d. of the unpredicted $\Delta \log(i/m)$ relative to the large firm

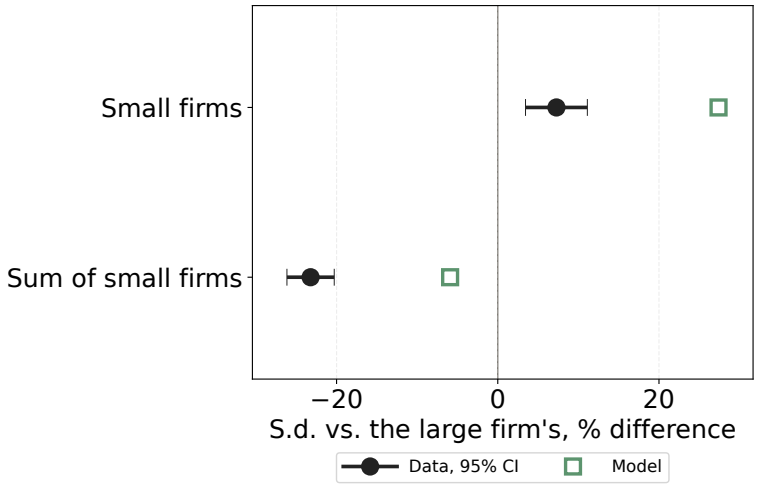


Figure 11 in Appendix D.4 shows the share of firms across i and m in the model and the data.

5 Quantitative Results

I now quantify the share of firm value due to reduced dependence on future external search. Relationships are reused when the investors and managers who remain are reassigned rather than returned to search. This reuse reduces dependence on future external search. As in Section 2, let $v(i, m, z)$ represent the joint surplus from keeping the firm together, which I think of as firm value. I decompose $v(i, m, z)$ to separate the role of relationships in reducing dependence on future search from other sources of firm value.

Definition 4. Decomposing Firm Value: *I decompose $v(i, m, z)$ into three components. Let k_j^* be the number of investors that a firm (i, m, z) with $m \geq 1$ currently assigns to its jth manager.*

Suppose the firm is split into m partitions, one per manager. Partition j is a firm $(k_j^*, 1, z_j)$ with one manager, its k_j^* investors, and the share $\tilde{f}(k_j^*)/\big(m\tilde{f}(i/m)\big)$ of the firm's Z, in proportion to the output it currently produces. Hence $z_j = \big(\tilde{f}(k_j^*)/(m\tilde{f}(i/m))\big)^{1-\nu} z$. The partitions together hold exactly the firm's Z, so the split creates no Z by copying it.

These partitions are separate from each other and cannot reassign investors or managers across them. The firm, unlike its partitions, can reassign capital across all its managers after future departures. However, each partition can still hire new managers, assign its investors to the new hires, and grow into a larger firm in the future. I call the first component of firm value reuse across partitions*. It is the value of future reassignment in one combined internal market relative to splitting the firm into these m partitions:*

$$\textit{Reuse across partitions} := v(i, m, z) - \sum_{j=1}^{m} v(k_j^*, 1, z_j).$$

Reusing relationships across partitions follows the logic of Proposition 1 and measures increasing returns to relationships.

Now, further suppose that the m partitions lose even the option to reassign capital to new managers. If they hire new managers, those new hires cannot be assigned any of the existing investors or use the partition's Z, which cannot be copied. Each partition can keep recruiting new investors because its existing manager can still service them. When its existing manager departs, the partition can no longer produce because it cannot reassign investors to a new manager. Therefore, its investors return to the external market. Let $w(k, z)$ denote the value of such a partition with k investors and z.

The value of the m partitions with the potential to reassign capital to new managers,

relative to the same partitions without that potential, is

$$\textit{Reuse within partitions} := \sum_{j=1}^{m} v(k_j^*, 1, z_j) - \sum_{j=1}^{m} w(k_j^*, z_j).$$

Reusing relationships within partitions is the value of being able to assign a lone manager's existing investors to new managers when they are recruited. I call the remaining value, $\sum_{j=1}^{m} w(k_j^*, z_j)$, other value. *This value comes from the current assignments, investor recruiting, and intangibles.*

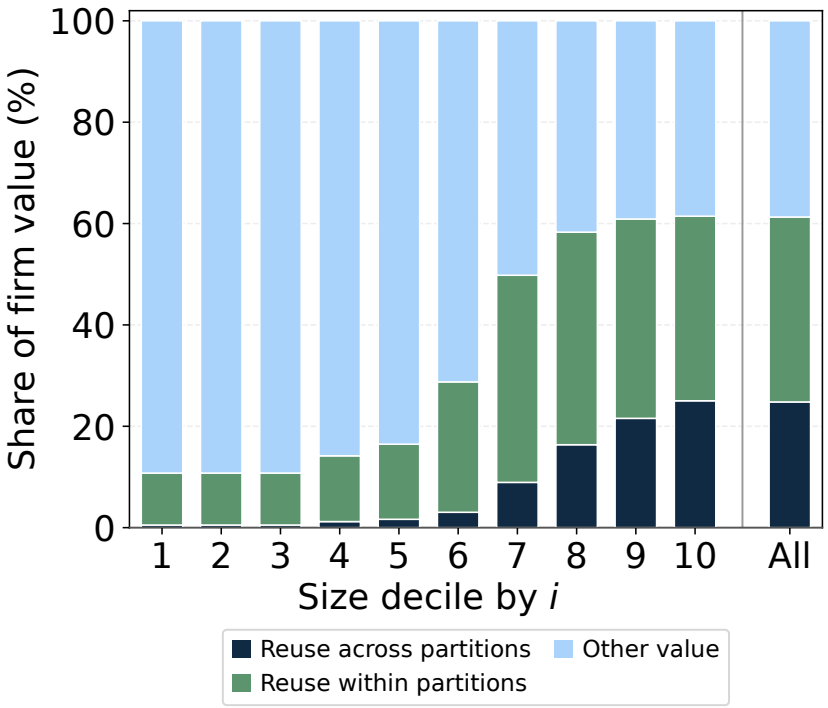
Reusing relationships within and across partitions captures the share of firm value attributable to reduced dependence on future external search. When the firm is split, the sum of the partitions' output equals the firm's output. Production and search have constant returns to scale, and the partitions hold exactly the firm's intangibles. Therefore, any difference in value comes entirely from reusing relationships.

Any exercise that breaks up a firm (i, m, z) takes its current size as given. But the firm formed its relationships through search, partly anticipating using them in the future. Therefore, the exercises below may understate the value of reusing relationships. However, the model has no agency problems inside the firm. Such problems could distort reassignment. Therefore, their absence means the exercises may overstate the value of reuse.

The share of value from a component is that component divided by firm value. Figure 2 decomposes firm value by size decile. It takes firms with $i, m > 0$ and creates deciles of i.[22] The bar labeled "All" pools these firms. Within each decile, it divides the sum of each component of Definition 4 by the decile's total firm value. Decile 1 corresponds to the smallest firms and decile 10 to the largest.

[22]Deciles 1 to 3 coincide because 36% of firms with $i, m > 0$ have $i = 1$.

Figure 2: Decomposing Firm Value by Size Decile



Identification of the pooled and decile-10 components: Figures 6 and 7 in Appendix D.2.

Pooled across firms, 61% (s.e. 1.0) of firm value comes from reusing relationships. Of this, 36 (s.e. 1.0) percentage points come from reuse within partitions and 25 (s.e. 1.9) from reuse across partitions. Appendix D.3 shows how these shares change with ϵ_I and ϵ_M. For the smallest firms, 11% of firm value comes from reusing relationships, almost all of it within partitions. For the largest firms, 61% of firm value comes from reusing relationships. Of this, 36 percentage points come from reuse within partitions. The remaining 25 (s.e. 2.0) percentage points come from reuse across partitions. Larger firms have more managers and thus more partitions to reassign across. Reuse across partitions is at most 3% of firm value in the six smallest deciles.

Appendix E.1 shows how the model infers these decompositions from the targeted moments, through each estimated parameter (Figure 12). A change in a targeted moment changes the estimated parameters and, through them, the decomposition. The year-1 investor leg, the mean of $\log(m)$, and the labor share move both shares the most (Figure 13). A one-standard-error rise in the year-1 investor leg raises the share of value from reuse within partitions by 0.79 standard errors and lowers the share of value from reuse across partitions by 0.78. A one-standard-error rise in the mean of $\log(m)$ raises the

first share by 0.58 and lowers the second by 0.58. A one-standard-error rise in the labor share raises the first share by 0.53 and lowers the second by 0.59. Each moment acts through several estimates at once. All three raise the estimate of α. The four estimates in Figure 12 each move the two shares in opposite directions, and α moves them the most. Section 5.1 studies why a lower α makes reusing relationships across partitions a larger part of firm value.

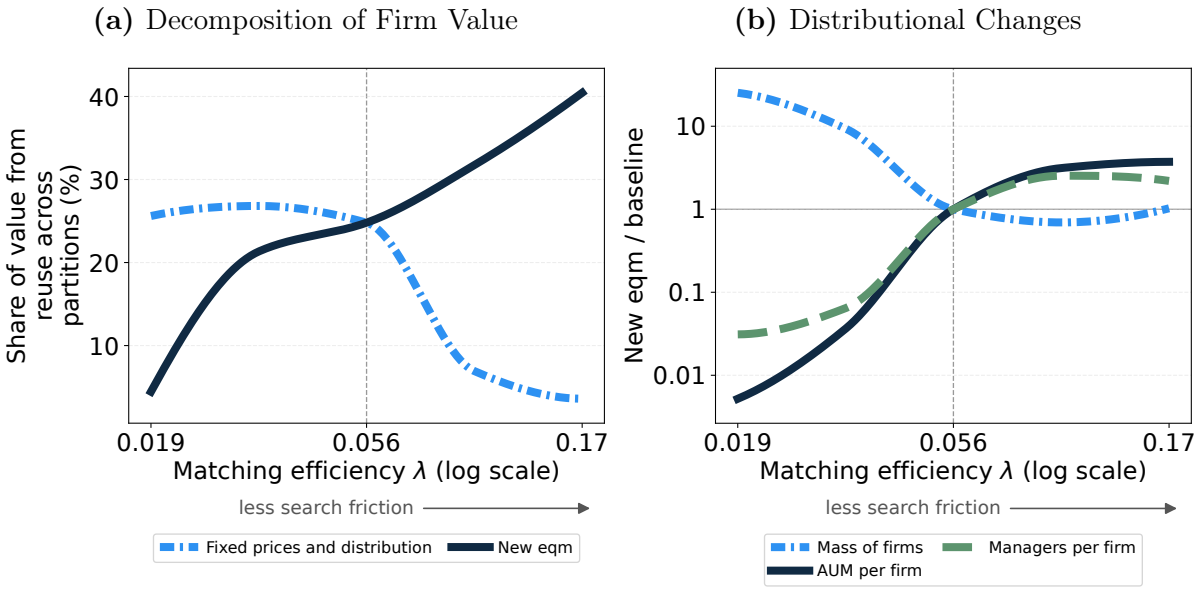
5.1 Technological Change

I now use the estimated model to ask how the share of firm value from reusing relationships across partitions responds to changes in technology in asset management. The first lowers search frictions, so investors and managers match faster without firms. The second lowers the weight on managers in production. Both changes are motivated by advances in artificial intelligence (AI). For example, AI agents of firms, investors, and managers may find counterparties, audit them, and negotiate terms. AI agents may also take on part of the investment work, enabling firms to produce the same output with fewer managers. I vary λ and α one at a time and do not model the source of either change. For a change in λ and for a large fall in α, the share of firm value from reuse across partitions moves in opposite directions in partial equilibrium, which holds market utilities fixed, and in full equilibrium.

Consider how much of a firm's value comes from reusing relationships across partitions, which measures increasing returns to relationships. Intuitively, this reuse of relationships may be a smaller part of firm value when search frictions are smaller. Reducing the need for future search might save less for investors and managers when frictions are lower. However, in equilibrium, the same frictions make it harder for the firm to grow large in the first place. Recall that, in the baseline theory, relationships lose all value for the firm as frictions vanish. At the other extreme, intuitively, firms cannot even form or grow at infinite frictions and cannot become valuable either.

Therefore, firm value depends on both how much investors and managers need a firm and how many relationships the firm can form. To separate these two forces, I re-solve the model at different λ in two ways. First, I solve the new equilibrium. Second, I fix both the distribution of firms and market utilities at the baseline and re-solve only the firm's HJB. Figure 3 shows the share of value from reuse across partitions, pooled across firms with $i, m > 0$.

Figure 3: Counterfactuals on Matching Efficiency



Lower search frictions reduce the share of value from reusing relationships across partitions when market utilities and the distribution are fixed but increase this share in equilibrium. At three times the matching efficiency, the new equilibrium has about the same mass of firms. The average investor base is 3.7 times as large and the average manager base is 2.2 times as large. Therefore, firms have more partitions to reassign across. The share of firm value from reuse across partitions is then 40%, against 25% at the baseline and 4% with market utilities and the distribution fixed. Conversely, at a third of the matching efficiency, the share rises to 26% with both fixed. However, it falls to 4% in equilibrium because almost every firm then has one manager and thus no partitions to reassign across.

Now, consider the consequences of a lower α. A lower α means that output is less sensitive to the number of managers. Therefore, firms may produce the same output with fewer managers. I do two exercises similar to the previous λ-exercises but with a key change. First, I solve the new equilibrium at different α. Second, I keep each new equilibrium's distribution of firms but fix market utilities at the baseline and re-solve only the firm's HJB. The second λ-exercise also fixes the distribution. This exercise fixes only market utilities because the strong equilibrium force here works through prices rather than the distribution. Figure 4 shows these exercises.

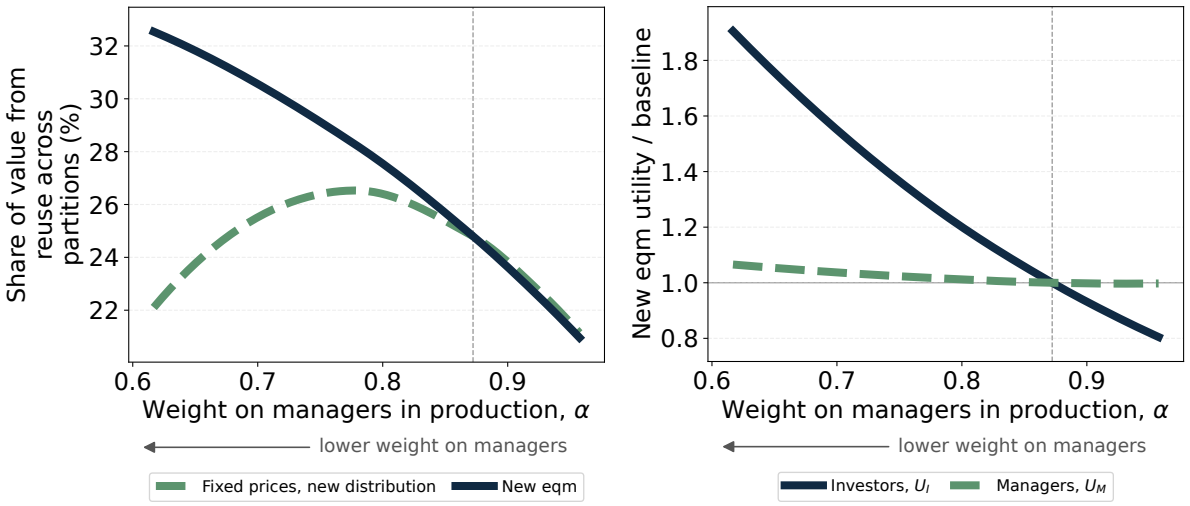
At fixed market utilities, a large fall in α reduces the share of firm value from reusing relationships across partitions because output is less sensitive to the number of managers.

Firms therefore have less need to search quickly for new managers, and relationships formed today save less future search. However, in equilibrium, a lower α actually increases this share. This discrepancy comes from how market utilities change. Investors' market utility increases. This means that, for any speed the firm chooses to grow its investor base, it faces a higher cost of inducing it. This means firms accept larger output losses from imbalance rather than pay the cost of repairing imbalances quickly.

Reusing relationships across partitions reduces dependence on future search. Future search is more expensive when α is lower. Therefore, in equilibrium, this reuse of relationships becomes an even larger part of firm value.

Figure 4: Counterfactuals on the Weight on Managers in Production

(a) Decomposition of Firm Value **(b)** Market utilities in the new equilibrium



Both changes make reusing relationships across partitions a larger part of firm value but through two different equilibrium forces. When search frictions fall, the distribution of firms changes as firms grow larger. When α falls, market utilities change and make search more expensive.

6 Conclusion

In this paper, I develop an equilibrium theory of why the relationships that make up asset management firms are valuable. In the model, investors and managers join a firm to reduce the need for future external search. This dynamic consideration makes firm value exhibit increasing returns to relationships.

In the model, the boundary of a firm is the set of investors and managers that it can reassign without new search. Expanding the boundary requires searching in the external market for more investors and managers that can then be integrated into the firm. Therefore, the model connects the value of relationships to firm dynamics. I estimate the model using data on entry, growth, and the size distribution of U.S. asset management firms.

The estimates attribute 61% of firm value to relationships reducing dependence on future external search and the remaining 39% to other sources, such as intangibles. Of the 61%, 25 percentage points come from increasing returns to relationships due to dynamic considerations. In counterfactuals, lower search frictions and a lower weight on managers in production increase the share of firm value from increasing returns to relationships. Both effects arise in full equilibrium. With market utilities and the distribution of firms fixed, the direction reverses for search frictions. In partial equilibrium, it reverses for sufficiently large reductions in this weight.

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Appendix

A Useful Objects

A.1 Matching Identities and Recruiting Costs

The constant-returns-to-scale property implies that the two matching rates satisfy

$$p_I(\theta) = \chi(\theta^{-1}, 1) = \frac{\chi(1,\theta)}{\theta} = \frac{p_M(\theta)}{\theta}.$$

For any positive investor recruiting rate μ_I, substitute $\theta_I = p_M^{-1}(\mu_I)$ into this identity to obtain

$$\frac{\mu_I}{p_I(p_M^{-1}(\mu_I))} = \frac{p_M(\theta_I)}{p_I(\theta_I)} = \theta_I = p_M^{-1}(\mu_I).$$

For any positive manager recruiting rate μ_M, the analogous substitution $\theta_M = p_I^{-1}(\mu_M)$ gives

$$\frac{\mu_M}{p_M(p_I^{-1}(\mu_M))} = \frac{p_I(\theta_M)}{p_M(\theta_M)} = \frac{1}{\theta_M} = \frac{1}{p_I^{-1}(\mu_M)}.$$

Multiplying these two expressions by $(r+\epsilon_I)U_I/\mu_I$ and $(r+\epsilon_M)U_M/\mu_M$, respectively, yields the costs to the firm $\bar{R}$ and $\bar{W}$. The reciprocal on the manager side reflects the definition of tightness as investors per manager. Investor recruiting uses θ_I investors per searching manager, while manager recruiting uses $1/\theta_M$ managers per searching investor.

A.2 Firm Problem Conditional on Market Utilities

Proposition 3. Equilibrium from the Firm's Perspective: *In a stationary symmetric equilibrium with positive entry, firm value solves*

$$\begin{aligned} r\, v(i,m) = \max_{\mu_I \geq 0,\, \mu_M \geq 0} \; & m\tilde{f}(i/m) - i(r+\epsilon_I)U_I - m(r+\epsilon_M)U_M \\ & + m\mu_I[v(i+1,m) - v(i,m) - \bar{R}(\mu_I; U_I)] \\ & + i\mu_M[v(i,m+1) - v(i,m) - \bar{W}(\mu_M; U_M)] \\ & + i\epsilon_I[v(i-1,m) - v(i,m)] \\ & + m\epsilon_M[v(i,m-1) - v(i,m)], \end{aligned}$$

where

$$\bar{R}(\mu; U_I) = \frac{(r+\epsilon_I)U_I\, p_M^{-1}(\mu)}{\mu}, \qquad \bar{W}(\mu; U_M) = \frac{(r+\epsilon_M)U_M}{\mu\, p_I^{-1}(\mu)}.$$

The entry market pins down the outside options as

$$\begin{aligned} U_I &= \frac{p_M'(\theta_E)}{r+\epsilon_I} v(1,1), \\ U_M &= \frac{p_M(\theta_E) - \theta_E p_M'(\theta_E)}{r+\epsilon_M} v(1,1). \end{aligned} \tag{Entry Prices}$$

If $\mu_I^*(i,m)$ and $\mu_M^*(i,m)$ denote the firms' optimal recruiting policies, entry tightness satisfies

$$\theta_E = \frac{I - \sum_{(i,m)} g(i,m)\left[i + m\, p_M^{-1}(\mu_I^*(i,m))\right]}{1 - \sum_{(i,m)} g(i,m)\left[m + i/p_I^{-1}(\mu_M^*(i,m))\right]}. \tag{Entry Tightness}$$

Proof. Fix the market utilities U_I and U_M. An individual incumbent is atomistic, so its recruiting choices do not change either market utility. Production net of current members' flow outside options is $m\tilde{f}(i/m) - i(r+\epsilon_I)U_I - m(r+\epsilon_M)U_M$. Each investor recruited at speed μ_I costs the firm $\bar{R}(\mu_I; U_I)$, and each manager recruited at speed μ_M costs it $\bar{W}(\mu_M; U_M)$. Investor and manager arrivals occur at rates $m\mu_I$ and $i\mu_M$. Relationship deaths occur at rates $i\epsilon_I$ and $m\epsilon_M$. These flow payoffs and transition values give the HJB in equation (Firm's HJB).

The conditional firm problem does not determine the market utilities. Positive entry and aggregate market clearing determine them in equilibrium. Let $\theta_E := p_M^{-1}(\mu_E^*)$ denote entry-submarket tightness. At a positive interior equilibrium μ_E^*, the first-order condition

for the entry maximization in equation (Entry Indifference) is

$$v(1,1) = \bar{R}(\mu_E^*; U_I) + \mu_E^* \bar{R}'(\mu_E^*; U_I) = U_I \frac{r + \epsilon_I}{p_M'(\theta_E)}.$$

Substituting this condition into the entry problem gives (Entry Prices).

The aggregate market-clearing conditions determine θ_E. Subtracting incumbent investor demand from the investor supply gives the investor mass in the entry submarket. The analogous manager residual gives its manager mass. Their ratio is entry tightness, which gives equation (Entry Tightness). Positive entry makes its denominator strictly positive. □

A.3 Payments Conditional on a Meeting

For any active investor-directed offer with speed μ,

$$\text{cost to the firm} = \bar{R}(\mu; U_I), \qquad \text{total investor payment} = U_I + \bar{R}(\mu; U_I).$$

For any active manager-directed offer with speed μ,

$$\text{cost to the firm} = \bar{W}(\mu; U_M), \qquad \text{total manager payment} = U_M + \bar{W}(\mu; U_M).$$

Since $\mu \bar{R}(\mu; U_I)$ is strictly convex and zero at zero, for any $\mu > 0$,

$$\bar{R}(\mu; U_I) < \bar{R}(\mu; U_I) + \mu \bar{R}'(\mu; U_I),$$

so $\bar{R}'(\mu; U_I) > 0$. The same holds for $\bar{W}$.

A.4 Stationary Distribution

The mass of firms $g(i, m)$ is defined at every $(i, m) \in \mathbb{Z}_+^2$ other than $(0, 0)$. States with no investors or no managers are firm states because the firm exits only at $(0, 0)$ and equation (Firm's HJB) holds at every other state.

A firm at (i, m) moves to $(i+1, m)$ at rate $m\mu_I^*(i, m)$ and to $(i, m+1)$ at rate $i\mu_M^*(i, m)$. It moves to $(i-1, m)$ at rate $i\epsilon_I$ and to $(i, m-1)$ at rate $m\epsilon_M$. These are the jump rates in equation (Firm's HJB) at the optimal recruiting rates. Stationarity requires that the flow of firms out of each firm state equal the flow of firms into it. The resulting balance conditions are the Kolmogorov forward equations for g.

At every state with $i \geq 1$ and $m \geq 1$ other than $(1,1)$, the balance condition is

$$\begin{aligned}\left[m\mu_I^*(i,m) + i\mu_M^*(i,m) + i\epsilon_I + m\epsilon_M\right] g(i,m) = {} & m\mu_I^*(i-1,m)\, g(i-1,m) \\ & + i\mu_M^*(i,m-1)\, g(i,m-1) \\ & + (i+1)\epsilon_I\, g(i+1,m) \\ & + (m+1)\epsilon_M\, g(i,m+1).\end{aligned}$$

The left-hand side is the outflow from (i,m). The first two terms on the right-hand side are recruiting by firms at $(i-1,m)$ and $(i,m-1)$. The last two are exits of members from firms at $(i+1,m)$ and $(i,m+1)$. If $i=0$ or $m=0$, the condition is the same without the term in $i-1$ or $m-1$, respectively.

Firms enter at $(1,1)$ at flow γ_E. Therefore, the balance condition at $(1,1)$ adds γ_E to its inflows,

$$\begin{aligned}\left[\mu_I^*(1,1) + \mu_M^*(1,1) + \epsilon_I + \epsilon_M\right] g(1,1) = {} & \gamma_E + \mu_I^*(0,1)\, g(0,1) + \mu_M^*(1,0)\, g(1,0) \\ & + 2\epsilon_I\, g(2,1) + 2\epsilon_M\, g(1,2).\end{aligned}$$

The inflow $\mu_I^*(0,1)\, g(0,1)$ is not part of the entry flow γ_E. A firm at $(0,1)$ faces the same problem as an unmatched manager who posts an entry offer, and Lemma 2 shows that $v(0,1)=0$. The firm nonetheless remains in g because its manager is in a firm and is therefore not unmatched. Its manager is not among the γ_E/μ_E^* managers who post entry offers. Both aggregate market-clearing conditions count its manager and the investors who search for its offer through the terms of their sums at $(i,m)=(0,1)$.

These conditions describe the baseline model.

B Theoretical Results

The results in this appendix are for the baseline model, without Z or free disposal. In the estimated model, Definition 4 measures the corresponding values numerically.

B.1 Strict Super-Additivity

Lemma 1. Weak Super-Additivity: *Under the maintained assumptions of the baseline model,*

$$v(i+i', m+m') \geq v(i,m) + v(i',m'),$$

for all $(i,m),(i',m') \in \mathbb{Z}_+^2 \setminus \{(0,0)\}$.

Proof of Lemma 1. Take two standalone firms. After combining them, keep their relationships labeled as partitions 1 and 2. Each partition keeps its own search and death clocks. Each successful search forms a relationship labeled by the partition that found the new counterparty.

Under the replication policy, recruiting costs are exactly the same as for the two standalone firms. At every date, the combined firm can give each manager the capital that manager has in its standalone firm and give any remaining capital to any of its managers. Because $\tilde{f}$ is weakly increasing and the best split produces at least as much as any other split,

$$(m_t^1 + m_t^2)\tilde{f}\left(\frac{i_t^1 + i_t^2}{m_t^1 + m_t^2}\right) \geq m_t^1 \tilde{f}(i_t^1/m_t^1) + m_t^2 \tilde{f}(i_t^2/m_t^2),$$

where a term with no managers is zero. The combined firm's flow payoff is therefore weakly larger on every path. Applying this construction to any policy path and taking the supremum over those policies gives

$$v(i + i', m + m') \geq v(i, m) + v(i', m'). \quad \square$$

Lemma 2. One-Sided Shells Are Exactly the Outside Options: *Under the maintained assumptions of the baseline model and positive entry,*

$$v(0, 1) = 0.$$

$$v(1, 0) = 0.$$

Proof of Lemma 2. First, consider a manager-only firm. Its HJB is

$$(r + \epsilon_M)v(0, 1) = -(r + \epsilon_M)U_M + \max_{\mu \geq 0} \left\{\mu\big[v(1, 1) - v(0, 1) - \bar{R}(\mu; U_I)\big]\right\}.$$

Separately, recall that the manager's entry condition is

$$(r + \epsilon_M)U_M = \max_{\mu \geq 0} \left\{\mu[v(1, 1) - \bar{R}(\mu; U_I)]\right\}.$$

If $v(0, 1) > 0$, the maximization in the firm's HJB is weakly below the entry maximization. Its right side is therefore weakly negative while its left side is strictly positive. If $v(0, 1) < 0$, evaluate the shell maximization at the positive optimizer of the entry problem. Its right side is then strictly positive while its left side is strictly negative. Hence $v(0, 1) = 0$.

Now consider an investor-only firm. The manager's entry problem, rewritten in terms

of tightnesses instead of match rates, is

$$(r+\epsilon_M)U_M = \max_{\theta>0}\left\{p_M(\theta)v(1,1) - (r+\epsilon_I)U_I\theta\right\}.$$

Therefore, for every $\theta > 0$,

$$\frac{p_M(\theta)v(1,1) - (r+\epsilon_M)U_M}{\theta} \leq (r+\epsilon_I)U_I.$$

The identity $p_I(\theta) = p_M(\theta)/\theta$ and the definition of $\bar{W}$ imply equality at the positive entry tightness. Thus

$$(r+\epsilon_I)U_I = \max_{\mu\geq 0}\left\{\mu[v(1,1) - \bar{W}(\mu; U_M)]\right\}.$$

The investor-only firm's HJB is

$$(r+\epsilon_I)v(1,0) = -(r+\epsilon_I)U_I + \max_{\mu\geq 0}\left\{\mu\big[v(1,1) - v(1,0) - \bar{W}(\mu; U_M)\big]\right\}.$$

The same argument gives $v(1,0) = 0$. □

Proof of Proposition 1. We already have weak super-additivity from Lemma 1. Now, we want to show strictness. Consider the two one-sided firms at states $(1,0)$ and $(0,1)$. Lemma 2 means that

$$v(1,0) + v(0,1) = 0.$$

Under the replication policy from Lemma 1, the combined firm starts at $(1,1)$. With positive probability, no jump occurs for a positive interval. The two partitions then produce zero while the combined firm produces $\tilde{f}(1) > 0$. Costs to the firm are weakly lower, and current-stock outside-option payments are exactly the same. Hence

$$v(1,1) > 0.$$

At $(0,1)$, recruiting the first investor has strictly positive marginal value at zero because $v(1,1) - v(0,1) > 0$ and $\bar{R}(0; U_I) = 0$. Therefore

$$\mu_I^*(0,1) > 0.$$

At $(1,0)$, the same argument gives

$$\mu_M^*(1,0) > 0.$$

Since $\epsilon_I, \epsilon_M > 0$, from any nonempty state, death shocks with no arrivals can leave one relationship on one side. At either one-sided state, the missing side arrives at a positive

rate. Another finite sequence of death shocks can then leave the other one-sided state. Thus every nonempty firm has positive probability of reaching either $(1,0)$ or $(0,1)$.

Apply the replication policy to the two standalone firms. Each partition keeps its own clocks. Since these clocks are independent, there is positive probability that one partition is at $(1,0)$ while the other is at $(0,1)$ and that neither partition jumps for a positive interval. Over this interval, the two standalone firms produce zero while the combined firm is at $(1,1)$ and produces $\tilde{f}(1) > 0$. Costs to the firm remain weakly lower, and current-stock outside-option payments remain exactly the same. The combined firm's discounted payoff is therefore weakly larger on every path and strictly larger with positive probability. Hence

$$v(i+i', m+m') > v(i,m) + v(i',m'). \quad \square$$

B.2 Labor Share

Proof of Corollary 1. Lemma 2 implies $v(0,1) = 0$. Proposition 1, applied to (i,m) and $(0,1)$, therefore gives

$$v(i, m+1) > v(i,m) + v(0,1) = v(i,m).$$

Thus adding one manager creates strictly positive marginal firm value,

$$v(i, m+1) - v(i,m) > 0.$$

Because $\bar{W}(0; U_M) = 0$, the derivative of the manager-recruiting objective at zero is strictly positive:

$$v(i, m+1) - v(i,m) - \bar{W}(0; U_M) = v(i, m+1) - v(i,m) > 0.$$

Hence $\mu_M^*(i,m) > 0$. The equilibrium recruiting speed is therefore interior and satisfies

$$v(i, m+1) - v(i,m) = \bar{W}(\mu_M^*(i,m); U_M) + \mu_M^*(i,m)\bar{W}'(\mu_M^*(i,m); U_M).$$

Since $\mu\bar{W}(\mu; U_M)$ is strictly convex and zero at zero, $\bar{W}' > 0$, so the first-order condition implies

$$\bar{W}(\mu_M^*(i,m); U_M) < v(i, m+1) - v(i,m).$$

The cost to the firm is therefore strictly smaller than the marginal firm value. Adding

U_M to both sides gives

$$U_M + \bar{W}(\mu_M^*(i,m); U_M) < U_M + v(i,m+1) - v(i,m).$$

Hence $l(i,m) < 1$. □

B.3 The Frictionless Limit

Under the scaled matching function in Definition 3, the costs to the firm are

$$\bar{R}_\lambda(\mu; U_I) = \frac{(r+\epsilon_I)U_I\, p_M^{-1}(\mu/\lambda)}{\mu}, \qquad \bar{W}_\lambda(\mu; U_M) = \frac{(r+\epsilon_M)U_M}{\mu\, p_I^{-1}(\mu/\lambda)}.$$

Lemma 3. Aggregate Firm Value Identity: *Define*

$$Y_\lambda := \sum_{i,m} g_\lambda(i,m) m \tilde{f}(i/m) \qquad \textit{and} \qquad FV_\lambda := \sum_{i,m} g_\lambda(i,m) v_\lambda(i,m).$$

In every equilibrium indexed by λ*,*

$$r\, FV_\lambda = Y_\lambda - (r+\epsilon_I)U_{I,\lambda} I - (r+\epsilon_M)U_{M,\lambda}.$$

Proof of Lemma 3. Let firm flow profits be

$$\pi_\lambda(i,m) := m\tilde{f}(i/m) - i(r+\epsilon_I)U_{I,\lambda} - m(r+\epsilon_M)U_{M,\lambda}$$
$$- m\mu_{I,\lambda}^*(i,m)\bar{R}_\lambda(\mu_{I,\lambda}^*(i,m); U_{I,\lambda}) - i\mu_{M,\lambda}^*(i,m)\bar{W}_\lambda(\mu_{M,\lambda}^*(i,m); U_{M,\lambda}).$$

The aggregate HJB is

$$r\,\mathrm{FV}_\lambda = \sum_{i,m} g_\lambda(i,m)\pi_\lambda(i,m) + \sum_{i,m} g_\lambda(i,m)(Q_\lambda v_\lambda)(i,m),$$

where Q_λ is the incumbent transition operator in economy λ.

Stationarity implies that incumbent transitions and entry have zero aggregate effect on the stationary distribution. Therefore

$$\sum_{i,m} g_\lambda(i,m)(Q_\lambda v_\lambda)(i,m) = -\gamma_{E,\lambda} v_\lambda(1,1).$$

Thus

$$r\,\mathrm{FV}_\lambda = \sum_{i,m} g_\lambda(i,m)\pi_\lambda(i,m) - \gamma_{E,\lambda} v_\lambda(1,1).$$

Aggregate investor market clearing implies

$$\begin{aligned}(r+\epsilon_I)U_{I,\lambda}I = &\sum_{i,m} g_\lambda(i,m)i(r+\epsilon_I)U_{I,\lambda} \\ &+ \sum_{i,m} g_\lambda(i,m)m\mu^*_{I,\lambda}(i,m)\bar{R}_\lambda(\mu^*_{I,\lambda}(i,m);U_{I,\lambda}) \\ &+ \gamma_{E,\lambda}\bar{R}_\lambda(\mu^*_{E,\lambda};U_{I,\lambda}).\end{aligned}$$

Aggregate manager market clearing and the entry condition imply

$$\begin{aligned}(r+\epsilon_M)U_{M,\lambda} = &\sum_{i,m} g_\lambda(i,m)m(r+\epsilon_M)U_{M,\lambda} \\ &+ \sum_{i,m} g_\lambda(i,m)i\mu^*_{M,\lambda}(i,m)\bar{W}_\lambda(\mu^*_{M,\lambda}(i,m);U_{M,\lambda}) \\ &+ \gamma_{E,\lambda}\left[v_\lambda(1,1) - \bar{R}_\lambda(\mu^*_{E,\lambda};U_{I,\lambda})\right].\end{aligned}$$

Adding gives

$$(r+\epsilon_I)U_{I,\lambda}I + (r+\epsilon_M)U_{M,\lambda} = Y_\lambda - \sum_{i,m} g_\lambda(i,m)\pi_\lambda(i,m) + \gamma_{E,\lambda}v_\lambda(1,1).$$

Substituting into the aggregate HJB gives

$$r\,\mathrm{FV}_\lambda = Y_\lambda - (r+\epsilon_I)U_{I,\lambda}I - (r+\epsilon_M)U_{M,\lambda}.$$

Therefore

$$\mathrm{FV}_\lambda = \frac{1}{r}\left[Y_\lambda - (r+\epsilon_I)U_{I,\lambda}I - (r+\epsilon_M)U_{M,\lambda}\right]. \quad \square$$

Proof of Proposition 2. Define

$$c_{I,\lambda}(\mu) := (r+\epsilon_I)p_M^{-1}(\mu/\lambda), \qquad c_{M,\lambda}(\mu) := \frac{r+\epsilon_M}{p_I^{-1}(\mu/\lambda)}.$$

Then $\bar{R}_\lambda(\mu;U_I) = U_I c_{I,\lambda}(\mu)/\mu$ and $\bar{W}_\lambda(\mu;U_M) = U_M c_{M,\lambda}(\mu)/\mu$.

First, we want to show that equilibrium outside options remain bounded. Write $Y_\lambda := \sum_{i,m} g_\lambda(i,m)m\tilde{f}(i/m)$ and $\mathrm{FV}_\lambda := \sum_{i,m} g_\lambda(i,m)v_\lambda(i,m)$. Joint concavity, constant returns to scale, weak monotonicity, and the aggregate market-clearing conditions imply $Y_\lambda \le \tilde{f}(I)$.

Then, combine this result with two older results. The option not to search implies that both outside options are non-negative. Repeated application of Lemma 1 and the zero shell values gives $v_\lambda(i,m) \ge 0$, so $\mathrm{FV}_\lambda \ge 0$. Lemma 3 shows that flow outside options

equal total surplus created less the return on holding all incumbent firms. Therefore,

$$0 \leq (r+\epsilon_I)U_{I,\lambda}I + (r+\epsilon_M)U_{M,\lambda} = Y_\lambda - r\,\mathrm{FV}_\lambda \leq Y_\lambda \leq \tilde{f}(I).$$

Hence $U_{I,\lambda} = O(1)$ and $U_{M,\lambda} = O(1)$.

Second, we want to show that the entrant firm value is $O(\lambda^{-1})$. Entry indifference requires

$$(r+\epsilon_M)U_{M,\lambda} = \max_{\mu_E \geq 0} \left\{\mu_E[v_\lambda(1,1) - \bar{R}_\lambda(\mu_E; U_{I,\lambda})]\right\}.$$

The left-hand side must be weakly greater than the surplus from entry evaluated at the possibly suboptimal rate $\mu_E = \lambda p_M(1)$. Recall that $p_M(1) > 0$ and $\bar{R}_\lambda(\lambda p_M(1); U_{I,\lambda}) = (r+\epsilon_I)U_{I,\lambda}/(\lambda p_M(1))$. Then, the inequality derived from entry can be rewritten as

$$0 \leq v_\lambda(1,1) \leq \frac{(r+\epsilon_M)U_{M,\lambda} + (r+\epsilon_I)U_{I,\lambda}}{\lambda p_M(1)} = O(\lambda^{-1}),$$

where the first weak inequality stems from strict super-additivity and the second one is due to the (possibly suboptimal) entry rate we postulated.

Third, we want to show that marginal firm value is $O(\lambda^{-1})$. Suppose first that $v_\lambda(i,m) = O(\lambda^{-1})$ for some $m \geq 1$ and any $i \geq 0$. Isolating investor recruiting in the firm's HJB gives

$$\max_{\mu_I \geq 0} \left\{m\mu_I[v_\lambda(i+1,m) - v_\lambda(i,m) - \bar{R}_\lambda(\mu_I; U_{I,\lambda})]\right\} \leq$$
$$(r + i\epsilon_I + m\epsilon_M)v_\lambda(i,m) + i(r+\epsilon_I)U_{I,\lambda} + m(r+\epsilon_M)U_{M,\lambda}.$$

The manager-recruiting maximum is non-negative. Since $m\tilde{f}(i/m) \geq 0$ and $v_\lambda \geq 0$, the inequality follows. Now evaluate the expression on the left at the possibly suboptimal recruiting rate $\mu_I = \lambda p_M(1)$. Using $\bar{R}_\lambda(\lambda p_M(1); U_{I,\lambda}) = (r+\epsilon_I)U_{I,\lambda}/(\lambda p_M(1))$ gives

$$0 \leq v_\lambda(i+1,m) - v_\lambda(i,m) \leq \frac{(r + i\epsilon_I + m\epsilon_M)v_\lambda(i,m) + i(r+\epsilon_I)U_{I,\lambda} + m(r+\epsilon_M)U_{M,\lambda}}{m\lambda p_M(1)}$$
$$+\frac{(r+\epsilon_I)U_{I,\lambda}}{\lambda p_M(1)} = O(\lambda^{-1}).$$

The first weak inequality follows from weak super-additivity. The last equality follows because outside options are $O(1)$ and, by hypothesis, $v_\lambda(i,m) = O(\lambda^{-1})$.

The same argument evaluated at $\mu_M = \lambda p_I(1)$, for which $c_{M,\lambda}(\lambda p_I(1)) = r+\epsilon_M$, gives, for $i \geq 1$,

$$0 \leq v_\lambda(i,m+1) - v_\lambda(i,m) = O(\lambda^{-1}).$$

By induction from $(1,1)$, $v_\lambda(i,m) = O(\lambda^{-1})$ for every fixed $i, m \geq 1$. On the axes, strict super-additivity and the zero shell values imply $0 \leq v_\lambda(i,0) < v_\lambda(i,1)$ for $i \geq 1$ and $0 \leq v_\lambda(0,m) < v_\lambda(1,m)$ for $m \geq 1$. □

C Data

C.1 Filings

Generally, firms have to register once their regulatory assets under management (RAUM) reach \$110M, or \$150M if they advise only private funds, and may stay registered until their RAUM falls below \$90M. RAUM is gross of liabilities and includes investors' uncalled commitments to private funds. Advisers only to venture capital funds are exempt and file a short report that omits RAUM and staff counts, so they are not in my data.

For each adviser and year, I keep the last filing submitted in that calendar year, so that each adviser contributes one observation per year. The links between advisers in Appendix C.2 come from every filing of the year. I use filings from 2018 to 2025 to compute moments because the form adopted in October 2017 is the first to report assets by client type and how separate accounts are invested. I use filings from 2015 to 2017 only to follow firms over time and filings back to 2000 only to date firms. The base years of the regressions are 2019 to 2023 because a regression needs a change into t and changes up to $t+2$.

C.2 Consolidation

I consolidate advisers cross-sectionally and in the time series. In a year, two advisers are grouped as a firm when each reports the other as controlling, controlled by, or under common control with it, and as sharing supervised persons, which is a box in Section 7.A of Schedule D. Chains of such pairs form one firm. Across years, if two groupings of advisers share more than half of each grouping's advisers in adjacent years, and a grouping with several advisers keeps one of its linked pairs, I consider that the continuation of a firm. A year without a filing ends the history. If two firms merge, both histories end and the merged firm starts a new one. It is not an entrant because its age comes from its oldest member. A merger or an acquisition brings in investors and managers who were already with another firm, which is not growth in the model. For changes over time, I therefore combine firms that share an adviser in adjacent years into one unit and sum

its active AUM and advisory staff in every year, including the parts outside the sample. A change over several years, as in the regressions from $t-1$ to $t+2$, uses the unit that links firms within all of those years. Each unit enters once, and it enters when it contains a firm-year of the sample in every year. A firm with unchanged advisers is its own unit. The sums leave out parts with zero active AUM or zero staff and parts before 2018. A unit's controls are the maximum over its firm-years at t.

C.3 Assets Under Management and Managers

Broadly, firms report RAUM in investment companies (e.g., mutual funds), business development companies (BDCs), private funds (e.g., hedge funds and private equity), and separately managed accounts (SMAs), which cover all else. Generally, the first three pool many different investors' claims, whereas an SMA is customized to each investor.

For SMAs, firms also break down the vehicles and asset types in which they manage those assets. I remove the SMA assets held in investment companies, BDCs, and other pooled vehicles because these assets are either already counted in the adviser's own funds or managed by another adviser. When a filing omits this breakdown, I impute the share held in pooled vehicles from the same firm's other filings (at least two from 2017 on), and otherwise from filers with the same largest asset type and RAUM quintile, pooled over the years from 2017 on, and otherwise from the mean of all filers.

For investment companies, I merge the funds each firm lists with N-CEN and CRSP data to identify passive investment vehicles and remove them from the firm's AUM. I match funds by Series ID, file number, and year. I apply the latest N-CEN index-fund box to all years, counting a blank as active, then the CRSP flags, then a name rule (INDEX, INDX, TRACKER, PASSIVE) that overrides an active label. Funds I cannot classify stay out of the firm's active share, which falls back to the share of the firm's year and business, then of the year, then to 100%. I remove passive funds because my theory focuses on active asset management that requires the effort and expertise of managers pursuing investments as opposed to a mechanical passive investment strategy. Separately managed accounts report their holdings but not whether they are passive, so I multiply the directly held SMA assets by the active share of the firm's funds. This share covers registered funds, private funds, and BDCs, and it is 100% for a firm with no classified funds.

Two advisers in the same firm can list the same fund, and the fund's assets then enter the firm's summed RAUM twice. To remove the repeat, I compute two totals over the

funds the firm's advisers list. The first adds a fund's assets once for each adviser that lists it. The second adds each fund's assets only once. I do this separately for registered and private funds and multiply the firm's RAUM in each by the ratio of the second total to the first.[23] A feeder fund counts as zero because its assets are counted in the master fund it invests in. Fund assets are net assets for registered funds and the largest reported gross asset value for private funds. A missing ratio takes the average of its cell and otherwise one. I leave BDCs unchanged, and they are never passive.

The RAUM net of passive funds, net of SMA assets held in pooled vehicles, net of funds counted twice within a firm, and after consolidation across related advisers is mapped to i. I call this measure active AUM.

The filers also report how many of their employees perform investment advisory functions. I map m to this count, summed over the firm's advisers. Advisers in one firm can also report the same employees. Each adviser reports its advisory staff and the advisory employees at each office other than its main office. When the office counts sum to more than the advisory staff, I scale them down to that total. I place the rest of its staff at its main office. When several advisers of a firm report staff at the same address, I keep the largest count because advisers that share personnel and an office report the same people. I multiply the firm's summed advisory staff by the ratio of the retained counts to the reported counts. The retained counts need not be whole numbers.

C.4 Entrants

A firm's age is the number of years since the earliest SEC filing among its advisers, and an entrant is a firm in that first year.

Moreover, a new adviser in its first year of filing is often not reported as under common control with an established firm even if it is. Therefore, a candidate entrant that first reports its link to an existing firm in the following year is merged into that firm in its first year. The merge follows chains of such links. A candidate entrant must also have at least \$90M of RAUM in its first year.

For firms that this rule makes entrants and that have more than \$500M of active AUM, I look up the founding year in public sources. Firms founded earlier or belonging to an established group are dated from that year and are not entrants because a first SEC filing dates a registration, not a firm. Firms found to be established without a known

[23] If two advisers list the same \$1B fund and a third lists a different \$1B fund, the first total is \$3B, the second is \$2B, and the firm's RAUM in that category is multiplied by 2/3.

founding year have an unknown age and do not set the median entrant in Appendix C.5.

C.5 Sample Construction

Before any of the steps above, I drop firm-years with zero advisory staff or zero active AUM because the moments are for firms with $i, m > 0$. I apply the following steps after those above, in this order. I drop a firm when fewer than half of its advisers have a main office in the United States in the firm's first year from 2018 on, whatever its RAUM that year. A missing office keeps the firm.

I keep only firm-years with at least \$90M of RAUM, the level below which an adviser must generally leave SEC registration. Below it, the firms that stay registered do so for other reasons and are not representative of small firms.

The sample keeps firms with directly held SMA assets below a third of active AUM in their first year in the sample, the first year from 2018 on with at least \$90M of RAUM. This cut removes firms that mainly run individual accounts, such as wealth managers. The firm stays in the sample in later years whatever its SMA share. I use only the first year for this test and the U.S. test because later years would select firms on how they change.

As $i = 1$ in the model is the entrant size, I normalize active AUM by its median among entrants, as defined in Appendix C.4, pooled over years. As $m = 1$ in the model is the entrant size, I normalize advisory staff by the interpolated median among the same firms because staff counts are integers, which makes the ordinary median jump when a single firm changes. I round staff to whole numbers before taking this median, I take both medians after all the cuts, and each bootstrap draw recomputes them. The median entrant has \$208M of active AUM and 4.6 advisory employees.

C.6 Labor Share

The target is the share of value added paid to labor among the largest employers of the sample. I take labor and value added from the segment tables in the annual reports of public parents of asset management firms for 2022 to 2025, or from the income statement of a parent that reports no segment outside asset management. Labor is all compensation of the asset management segments, including equity-based compensation, which I add back where a segment measure excludes it, and value added is their fee and realized performance revenue less non-labor expense. I drop a parent-year when the filing does not report this compensation (65 rows) because labor is then unobserved. Non-labor

expense excludes amortization and impairment of intangibles, deal and restructuring charges, interest, and taxes. When a filing does not report a realized basis, I keep only accrued performance revenue (8 rows). I exclude income from the parent's own investments and the compensation the filing ties to it. Where the filing does not separate the compensation tied to that income, I keep both the income and the compensation (20 rows) because removing the income alone would overstate the labor share. If the asset management segment is commingled with wealth management, I use that segment. Where the filing does not divide the compensation between the two businesses, I keep the revenue of both.

I link a parent to a firm in a year when one of the firm's Form ADV filings that year reports the parent as a control person, through a chain of ownership links of at least 50% or an equivalent managing role. Chains through a bank or trust company acting as trustee do not count. A firm linked to several parents with usable financials goes to the parent with the most linked filings, and a tie leaves the firm unlinked. I spread each parent's labor and value added over all of its linked firms, including firms dropped from the sample for their SMA share, in proportion to active AUM. The sample keeps the part on its own firms, so a small firm in the sample does not carry a whole parent.

In each year, I rank firms by advisory staff and keep the top 10%. The firm at the cutoff counts in proportion to its part above the cutoff, so the top 10% holds exactly a tenth of the firms, as in the model. Linked firms hold only a quarter to a third of the active AUM at the top. I therefore split the top 10% into rank bands of 0–0.5%, 0.5–1%, 1–2%, 2–5%, and 5–10%. Within each band, pooled over years, I compute labor and value added per dollar of active AUM among linked firms and multiply both by the band's total active AUM. A band without linked firms is merged into the band above it, and the top band into the band below it. The target is the sum of this labor over bands divided by the sum of this value added. Each bootstrap draw recomputes the ranks and bands and holds the parent links and financials fixed. The model counterpart is manager pay, including payments at the match, divided by output net of investor pay, also including payments at the match, over the top 10% of the stationary distribution ranked by m, with the same cutoff rule. Output net of investor pay is the model's counterpart of value added because the model has no non-labor inputs and the returns investors keep never enter a firm's revenue in the data.

Over these firms, the model's labor share is

$$\frac{\sum g(i,m,z)\left[m(r+\epsilon_M)U_M + i\,\mu_M^*\,\bar{W}(\mu_M^*;U_M)\right]}{\sum g(i,m,z)\left[z\big(m\tilde{f}(i/m)\big)^{\nu} - i(r+\epsilon_I)U_I - m\,\mu_I^*\,\bar{R}(\mu_I^*;U_I)\right]}.$$

C.7 Winsorization and Residualization

The steps run in this order. First, within each year of the data, I winsorize $\log(i)$ and $\log(m)$ at their 2.5th and 97.5th percentiles. In the model, I winsorize the stationary distribution. Second, I winsorize the changes of $\log(i)$ and $\log(m)$ from $t-1$ to t, from t to $t+1$, and from t to $t+2$ the same way, within base year, computing them from levels that are not winsorized. The model has no calendar years, and I winsorize its changes at the percentiles of the simulated firms pooled. Third, I residualize. The means of $\log(i)$ and $\log(m)$ use only the first step. The standard deviation of $\log(m)$ uses only the first step and year indicators. The regressions and the dispersion of $\Delta\log(m)$ use the second and the third. The standard deviation of $\log(i/m)$ uses the first and the third. Entry, the entrant shares, the top-decile sizes, and the labor share use none.

The business-mix index predicts a firm's $\log(i) - \log(m)$ from the values at date t of its business-line indicators, the mix of its active AUM across vehicles, its private funds' strategies, investors, and structure, its clients, services, and related persons, and its passive share of AUM. It contains no direct measure of size or staff, although the shares of employees with each registration, the counts of owners and related persons, and the log of the minimum investment correlate with size. I fit it by gradient boosting on four fifths of the firms and predict the other fifth, five times, so a firm's own ratio never enters its prediction, and I demean it by year.[24] For the regressions, I fit the same model separately to each regression variable, including scale and the changes, and subtract its prediction.

The controls contain no direct measure of the firm's size, ratio, age, history, or fees. Each bootstrap draw repeats the residualization, including the gradient-boosting fits. I residualize $(\log(i) + \log(m))/2$ the same way in the regressions.

[24] The outcome is demeaned by year before fitting. The model is histogram gradient boosting with 400 iterations, a learning rate of 0.05, 31 leaves, a minimum leaf of 40, and an ℓ_2 penalty of 1. The five folds are grouped by firm. Rows outside the level sample take the firm's average features over all years.

Table 5: Variance Removed by Residualization

	S.d. after	% of variance removed by		
Variable	**winsorization**	**Year indicators**	**Business mix**	**Total**
Dynamic regressions (firms observed from $t-1$ to $t+2$)				
$\log(i_t/m_t)$	1.097	0.3	33.0	33.3
$\log(i_t\, m_t)$	2.662	0.0	66.0	66.1
$\Delta \log(i_t/m_t)$	0.335	4.4	0.6	5.0
$\Delta \log(i_t\, m_t)$	0.422	3.3	3.4	6.7
$\Delta \log(i_{t+1})$	0.305	4.8	1.7	6.6
$\Delta \log(m_{t+1})$	0.191	0.4	0.1	0.5
$\Delta \log(i_{t+2})$	0.463	3.2	0.1	3.3
$\Delta \log(m_{t+2})$	0.293	0.3	−4.4	−4.1
Dispersion moments (all firm-years)				
$\log(i/m)$	1.085	0.3	32.1	32.4
$\log(m)$	1.100	0.0	0.0	0.0

Each share is the fraction of a variable's variance, pooled over firm-years, that a step removes after the winsorization its moments use. Year indicators are removed first, and the two shares add up to the total. Because a firm's own values never enter its business-mix prediction, that prediction can do worse than the year mean, and the business-mix share is then negative.

C.8 Moments and Bootstrap

I compute $\mathbb{E}\log(i)$, $\mathbb{E}\log(m)$, the entrant shares, and the standard deviations of $\log(m)$ and $\log(i/m)$ within each year and average them over 2018 to 2025 with equal weights. The top-decile mean and the regressions pool firm-years.

The top-decile target is the mean of $\log(i)$ over the top 10% of firms by $\log(i)$ within each year, pooled over years, among firm-years with $m \geq 1$, without winsorizing or residualizing. The firm at the cutoff counts in proportion to its part above the cutoff. In the model, I rank firms with $i, m > 0$ by i with the same rule.

The regressions pool base years 2019 to 2023 for the units present from $t-1$ to $t+2$. Each is a least-squares regression of a residualized change on an intercept, residualized $\log(i_t)$ and $\log(m_t)$, and their past changes.

$\widehat{\Omega}$ comes from 1,000 multinomial draws of the 5,485 firm histories, with the same draws for every moment and without shrinkage. In each draw, I recompute entrant units, winsorizations, residualizations, and other results. Each draw holds fixed the consolidation, the continuity of firms, sample membership, founding years, the merges of late-reporting

entrants, and the labor-share links and financials.

D Quantification

Table 6: Estimates of the Model's Parameters

No.	Parameter	Estimate	S.E.
1	λ	0.056	0.003
2	α	0.872	0.038
3	$\log(I)$	3.434	0.078
4	s_Z	3.744	0.072
5	ν	0.853	0.012
6	σ_Z	0.115	0.012
7	ζ	1.140	0.047

D.1 Firm's Problem

The firm's value $v(i, m, z)$ solves

$$
\begin{aligned}
0 = \max\Big\{ & - r\, v(i,m,z) + \max_{\mu_I \geq 0,\, \mu_M \geq 0} \Big[z\big(m\tilde{f}(i/m)\big)^{\nu} \\
& - i(r+\epsilon_I)U_I - m(r+\epsilon_M)U_M \\
& + m\,\mu_I \left[v(i+1,m,z) - v(i,m,z) - \bar{R}(\mu_I; U_I)\right] \\
& + i\,\mu_M \left[v(i,m+1,z) - v(i,m,z) - \bar{W}(\mu_M; U_M)\right] \\
& + i\,\epsilon_I \left[v(i-1,m,z) - v(i,m,z)\right] \\
& + m\,\epsilon_M \left[v(i,m-1,z) - v(i,m,z)\right] \\
& + \mu_z\, z\, \frac{\partial v(i,m,z)}{\partial z} + \frac{\sigma_z^2}{2}\, z^2\, \frac{\partial^2 v(i,m,z)}{\partial z^2} \Big], \\
& \max_{i' \leq i,\, m' \leq m,\, i'+m' < i+m} v(i',m',z) - v(i,m,z) \Big\},
\end{aligned}
\qquad \text{(Firm's HJB with } Z\text{)}
$$

with $v(0,0,z) = 0$, where $\sigma_z := (1-\nu)\sigma_Z$ is the volatility of $\log(z)$ and $\mu_z := (1-\nu)(\mu_Z - \sigma_Z^2/2) + \sigma_z^2/2$ is the drift of z.

The entry value in equation (Entry Prices) is the average of $v(1,1,z)$ over the entry distribution of z.

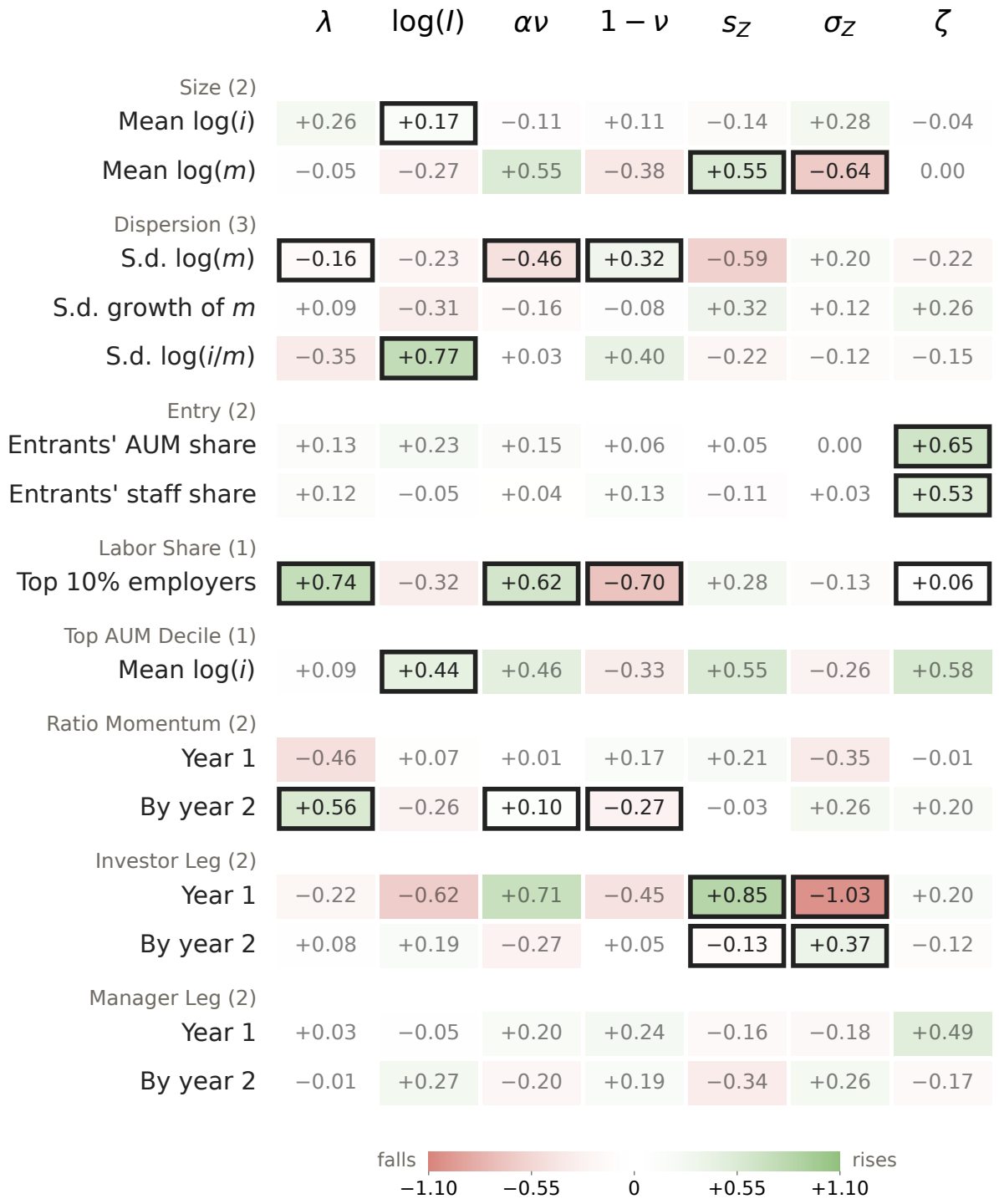
D.2 Identification

Table 7: Groups of Targeted Moments for Each Estimate

Parameter	Group 1	Group 2	Group 3
λ	S.d. $\log(m)$, labor share, momentum by year 2: 307%	Mean $\log(i)$, mean $\log(m)$, s.d. growth of m: 47%	S.d. $\log(i/m)$, year-1 momentum, year-1 investor leg: 28%
$\log(I)$	Mean $\log(i)$, s.d. $\log(i/m)$, top-decile mean $\log(i)$: 586%	Mean $\log(m)$, year-1 investor leg, investor leg by year 2: 102%	S.d. $\log(m)$, labor share, momentum by year 2: 41%
$\alpha\nu$	S.d. $\log(m)$, labor share, momentum by year 2: 302%	Mean $\log(m)$, year-1 investor leg, investor leg by year 2: 155%	Top-decile mean $\log(i)$, year-1 manager leg, manager leg by year 2: 48%
$1-\nu$	S.d. $\log(m)$, labor share, momentum by year 2: 306%	Mean $\log(i)$, s.d. $\log(i/m)$, top-decile mean $\log(i)$: 168%	Mean $\log(m)$, year-1 investor leg, investor leg by year 2: 103%
s_Z	Mean $\log(m)$, year-1 investor leg, investor leg by year 2: 212%	S.d. $\log(m)$, labor share, momentum by year 2: 199%	Top-decile mean $\log(i)$, year-1 manager leg, manager leg by year 2: 194%
σ_Z	Mean $\log(m)$, year-1 investor leg, investor leg by year 2: 230%	Mean $\log(i)$, s.d. growth of m, labor share: 73%	Top-decile mean $\log(i)$, year-1 manager leg, manager leg by year 2: 29%
ζ	Entrants' AUM share, entrants' staff share, labor share: 226%	S.d. growth of m, s.d. $\log(i/m)$, momentum by year 2: 63%	Top-decile mean $\log(i)$, year-1 investor leg, investor leg by year 2: 28%

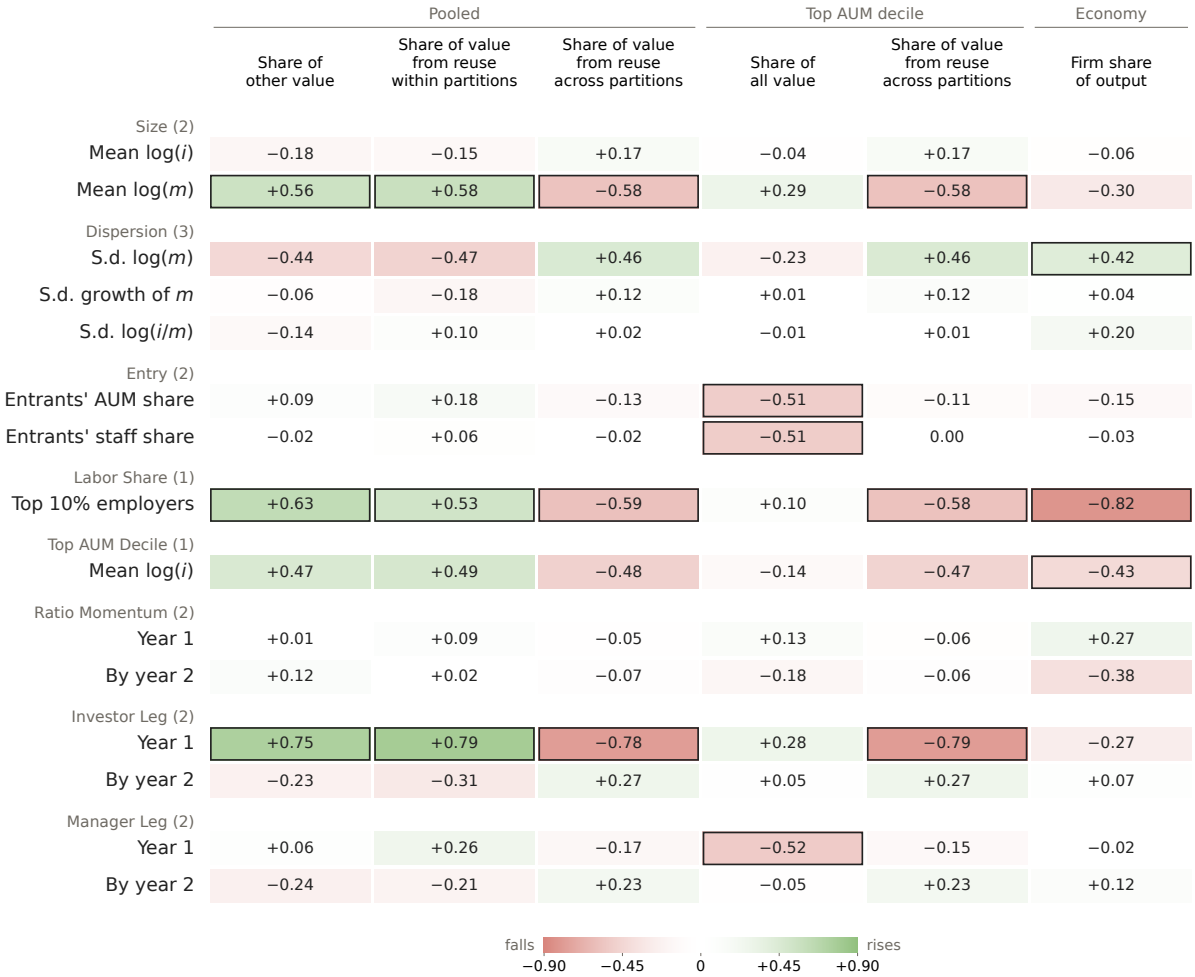
Note: Following Honoré et al. (2020), each cell lists three moments and the percentage rise in the parameter's standard error when they are dropped together and the efficient weighting matrix is recomputed without them. This rise is not the sum of the single-moment rises. Group 1 has the largest rise among all groups of three moments. Groups 2 and 3 repeat this step among the moments that the parameter's earlier groups do not use. A moment can appear for several parameters. A moment by year 2 is its year-1 coefficient plus its year-2 increment from Section 4.3.

Figure 5: Sensitivity of the Estimates to the Targeted Moments



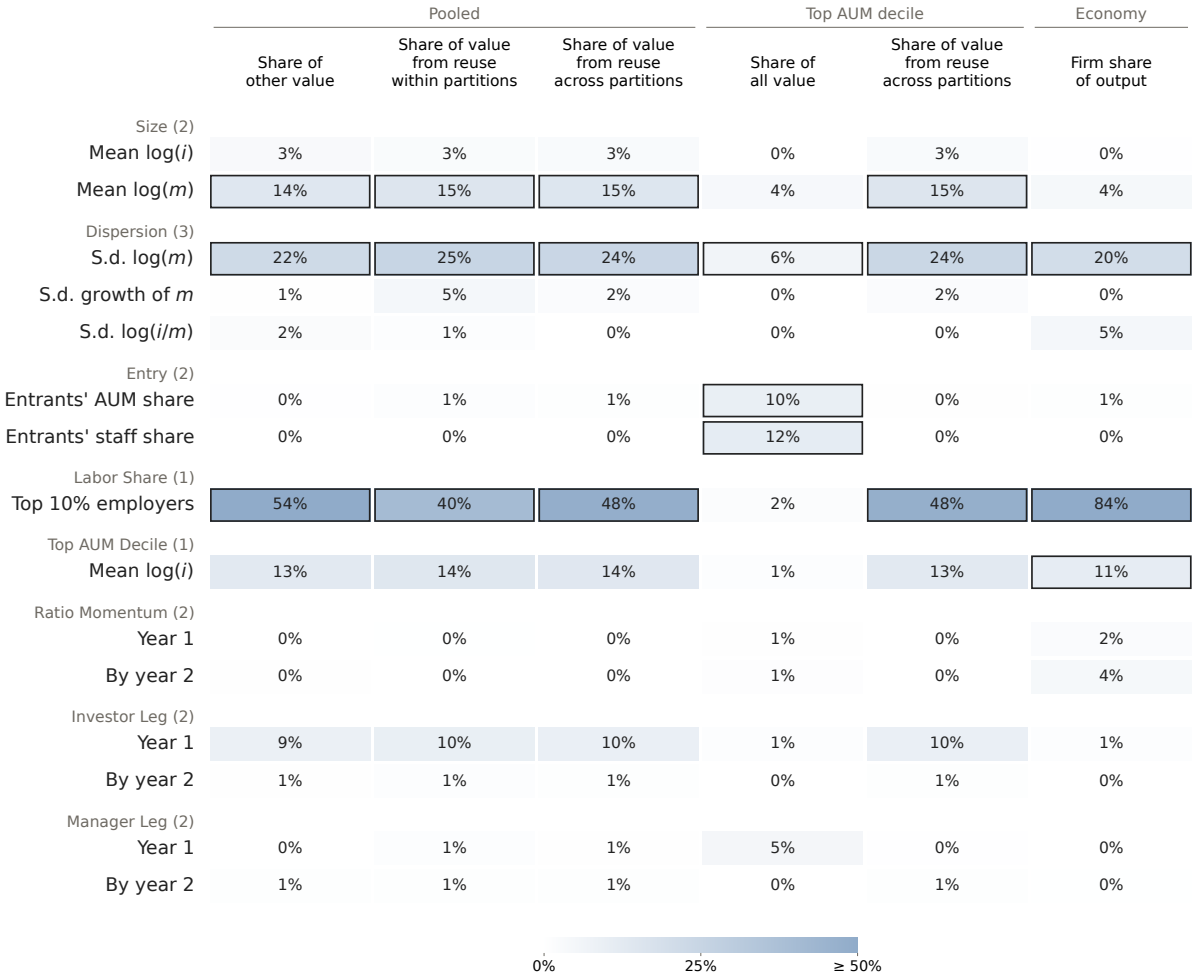
Note: Following Andrews et al. (2017), each cell is the change in the parameter, in its own standard errors, when the moment rises by one standard error. The framed cells are each parameter's Group 1 in Table 7. The other cells are in lighter text.

Figure 6: Sensitivity of the Components of Firm Value to the Targeted Moments



Note: As in Figure 5, for the components of Definition 4, pooled and in the top decile by i, each in its own delta-method standard errors. Firms with $i, m > 0$. Framed: each result's three largest cells.

Figure 7: Informativeness of the Targeted Moments for the Components of Firm Value



Note: Following Honoré et al. (2020), each cell is the percentage rise in the delta-method standard error of the component in Figure 6 when the moment is dropped and the efficient weighting matrix is recomputed without it. Firms with $i, m > 0$. Framed: each result's three largest cells.

D.3 Externally Calibrated Separation Hazards

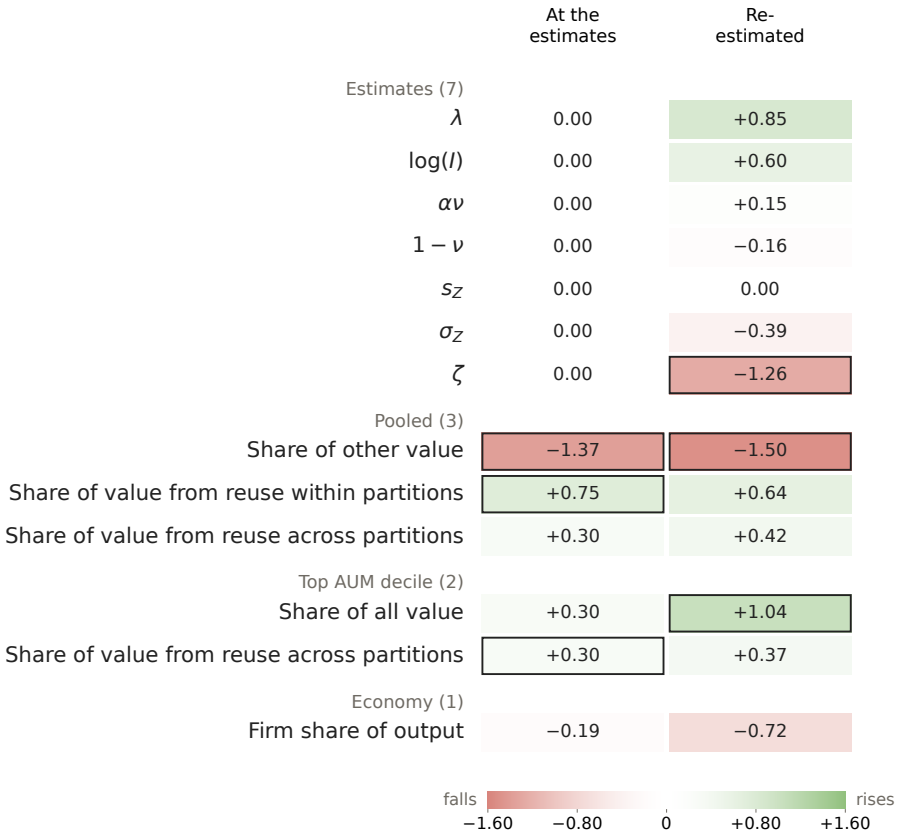
The data do not reveal how many investors and managers leave a firm. Therefore, the estimates condition on the values of ϵ_I and ϵ_M in Table 2. This subsection reports how the estimates and the results change with these values.

For ϵ_M, I obtain the manager tenure from the career-transition statistics in Jackson (2025), which imply manager-engagement hazards around 0.16 to 0.18 per year. I set $\epsilon_M = 0.18$. For ϵ_I, I obtain the LP-GP stickiness from the follow-on reinvestment statistics in Sensoy et al. (2013). The implied follow-on reinvestment rate is 0.743. With a 4.5-year fundraising interval, this implies $-\log(0.743)/4.5 = 0.066$. Therefore, I set $\epsilon_I = 0.065$.

In the following robustness exercises, I scale ϵ_I and ϵ_M by a common factor. Changing

this factor shifts the model's targeted moments, as a change of the opposite sign in the data moments would. Therefore, to first order, the estimates change by the sensitivity of Andrews et al. (2017) in Figure 5 applied to the opposite of the change in the model's moments. Figure 8 reports the change in the estimates and in the components of firm value when both hazards rise by 10%.

Figure 8: Sensitivity of the Estimates and the Components of Firm Value to the Separation Hazards

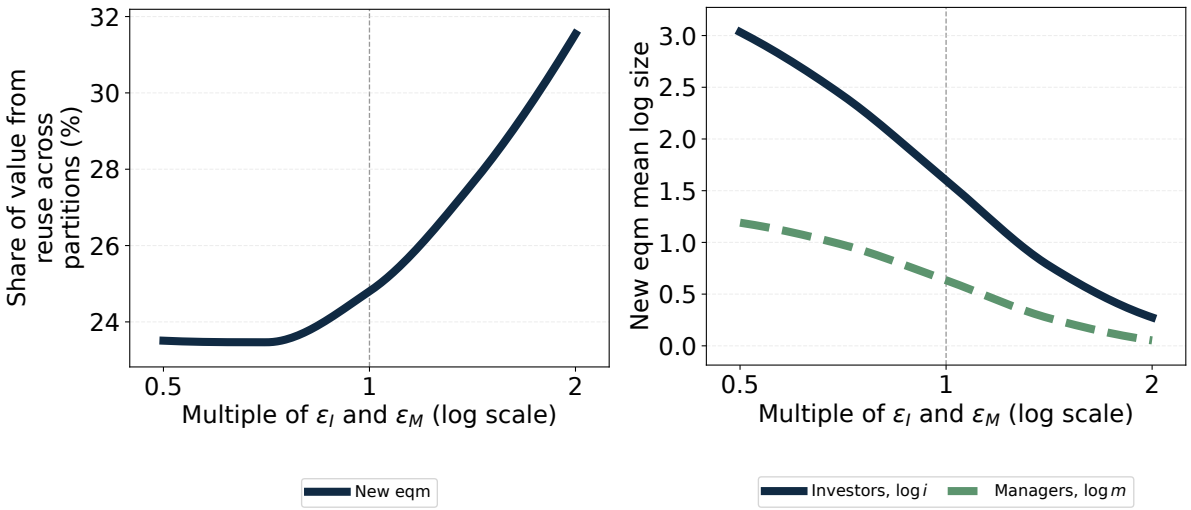


Note: Each cell is the first-order change, in its own standard errors, when ϵ_I and ϵ_M both rise by 10%. The first column holds the other parameters at their estimates. The second projects their re-estimation on the same targets to first order. The components of firm value are those of Figure 6. Firms with $i, m > 0$. Framed: each column's three largest cells.

Figure 9 re-solves the model with ϵ_I and ϵ_M scaled by a common factor from one half to two, with the other parameters at their estimates, in full equilibrium. Mean size is winsorized as the targets are.

Figure 9: Separation Hazards and Reuse across Partitions

(a) Reuse across partitions **(b)** Mean size



D.4 Untargeted Moments

Figure 1. Both panels use the firms observed from $t-1$ to $t+2$, as the dynamic regressions do. I winsorize the changes of $\log(i)$ and $\log(m)$, but not their levels, at their 2.5th and 97.5th percentiles within each year. Then, I residualize $\log(i)$, $\log(m)$, and their changes on business mix and year indicators as for the dynamic regressions (Appendix C.7). I add back the sample means of $\log(i)$ and $\log(m)$ and treat their exponentials as the firm's i and m, in units of the median entrant. In the model, I apply the same steps without residualizing because the model has no business mix. Because the model also has no years, I winsorize the changes at the pooled percentiles of the simulated firms, I demean each date's levels by these firms' means, and one pooled regression gives the conditional expectations.

In Figure 1a, small firms are the three quarters of firms with the lowest $\log(i\,m)$ at $t-1$, among firms with more than the median entrant's advisory staff. Investor-heavy firms are the quarter of small firms with the highest $\log(i/m)$ at $t-1$, and manager-heavy firms are the quarter with the lowest. I pair every investor-heavy firm with every manager-heavy firm of the same year. At each date, I sum the two firms' i and m. The large firm at the combined size averages the change in $\log(i/m)$ of firms with more than the median entrant's advisory staff at t. It weights them by a Gaussian kernel with bandwidth 0.10 in $\log(i)$ and $\log(m)$ around the sum's values at t. I weight each pair by the total kernel weight of these firms, for both the large firm and the sum. This panel

reports changes from t to $t+2$ without subtracting conditional expectations.

In Figure 1b, every firm is a large firm. Its small firms are firms with more than the median entrant's AUM and advisory staff at t, weighted by the same kernel around the large firm's $\log(i) - \log(2)$ and $\log(m) - \log(2)$ at t. I weight each large firm by the total kernel weight of its small firms. The sum adds the i and m of two distinct small firms of the same large firm. For every firm, I subtract from the change in $\log(i/m)$ from t to $t+2$ its conditional expectation from the dynamic regressions. For the sum, I evaluate this expectation at its own scale, ratio, and their past changes, with the real firms' coefficients. This panel reports each standard deviation relative to the large firm's. Figure 10 reports the levels. I truncate the kernel at 0.4 in each coordinate. For the first panel, I compute it on a grid with steps of 0.025. For the second, I average over every pair of a large firm's small firms when it has at most 400, and otherwise over 10^8 sampled pairs in total.

Figure 10: Untargeted: S.d. of $\Delta \log(i/m)$ after Shocks, in Levels

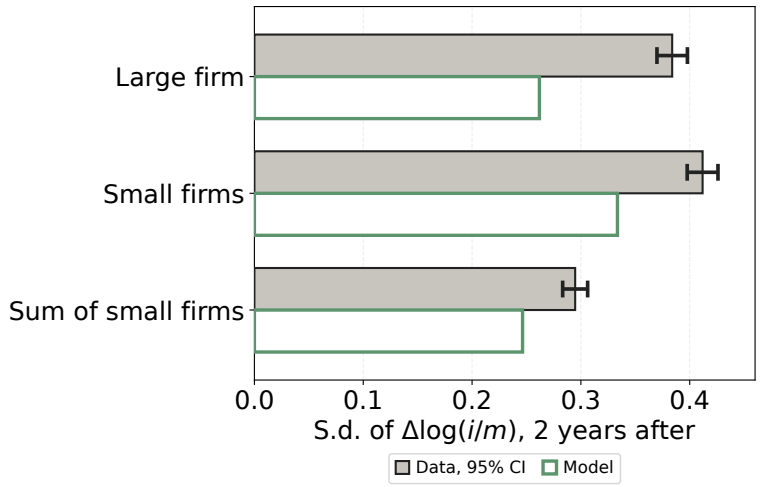
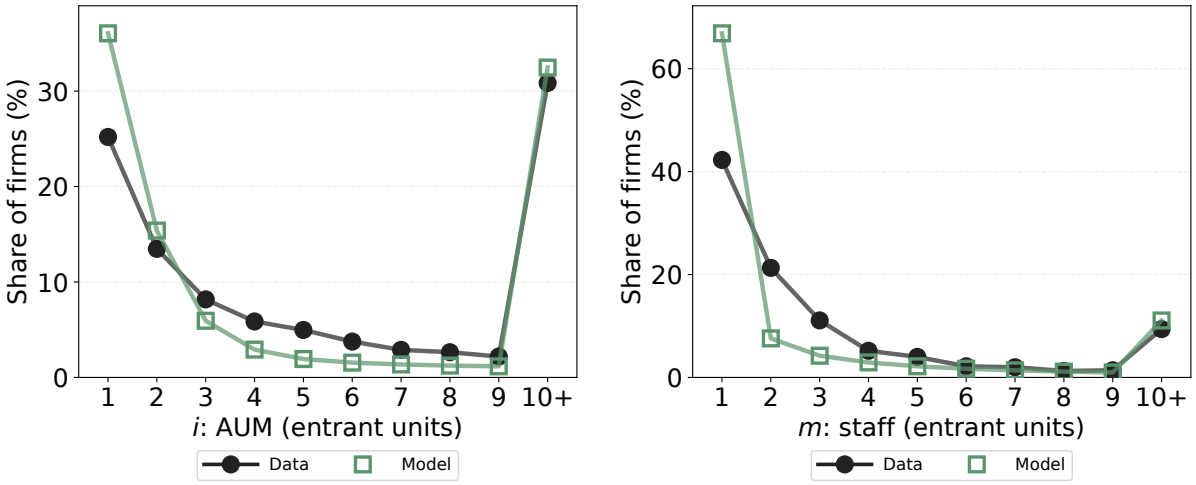


Figure 11. In the data, I round each firm's AUM and advisory staff to the nearest multiple of the median entrant's. The model counts investors and managers in that unit, among firms with $i, m > 0$. The estimation targets only the means of $\log(i)$ and $\log(m)$, the top-decile mean of $\log(i)$, and the standard deviations of $\log(m)$ and $\log(i/m)$, not these shares. The model puts 32% of firms at ten or more entrants' AUM, against 31%. It puts too many firms at one entrant's AUM, 36% against 25%. Relative to the data, it puts too many firms at a single manager, 67% against 42%, mostly at the expense of firms with two or three managers. The share of firms in each class from four managers up, including ten or more, is within four percentage points of the data.

Figure 11: Untargeted: Firm Size Distribution

(a) Investors (i) **(b)** Managers (m)



E Quantitative Results

E.1 Inferring Value

How does the model infer the value decompositions? Changes in the economy's parameters affect both the targeted moments and the value decompositions. In this way, a change in a targeted moment changes the estimated parameters and, through them, the decomposition.

To first order, the targeted moments move the estimates by the sensitivity of Andrews et al. (2017). Recall that Θ is the vector of estimated parameters, with kth element Θ_k, and $\widehat{\psi}$ is the vector of targeted data moments, with jth element $\widehat{\psi}_j$. Let $S(\Theta)$ denote a share of firm value in the equilibrium at Θ, for example the pooled share of value from reuse within partitions. For the estimator in Section 4.4, the chain rule gives

$$\frac{d\,S(\widehat{\Theta})}{d\,\widehat{\psi}_j} = \sum_k \frac{\partial\,\widehat{\Theta}_k}{\partial\,\widehat{\psi}_j}\,\frac{\partial\,S(\widehat{\Theta})}{\partial\,\Theta_k},$$

where $\widehat{\Theta}_k$ is the kth element of $\widehat{\Theta}$ and the estimated parameters depend on the moments, locally, as

$$\frac{\partial\,\widehat{\Theta}}{\partial\,\widehat{\psi}'} = \left(\frac{\partial M(\widehat{\Theta})'}{\partial\Theta}\,\widehat{\Omega}^{-1}\,\frac{\partial M(\widehat{\Theta})}{\partial\Theta'}\right)^{-1}\frac{\partial M(\widehat{\Theta})'}{\partial\Theta}\,\widehat{\Omega}^{-1}.$$

I report this total derivative in Figure 12, normalizing each object by its standard error so that different parameters and moments can be compared more easily. In this subsection, I report the effects of the three most informative moments through four key parameters.[25]

Smaller search frictions (i.e., higher λ) increase the share of value from reusing relationships. The share from reuse across partitions rises, and the share from reuse within partitions falls by less. This is an equilibrium effect that I emphasize in the first exercise of Section 5.1, presented in Figure 3. Easier search lets firms grow larger at the time of decomposition and also expect to grow larger in the future, so they have more future options to reassign across partitions, even as the external market's disadvantage relative to the nexus is partly relieved.

A larger labor share of income (among the top 10% of employers) suggests higher λ. This channel lowers the share of value from reuse within partitions by 0.15 standard errors (i.e., 0.74×0.20) and raises the share of value from reuse across partitions by 0.15 (i.e., 0.74×0.20). A less negative ratio momentum by year 2 also suggests higher λ. This channel lowers the first share by 0.11 (i.e., 0.56×0.20) and raises the second by 0.11 (i.e., 0.56×0.20). Other channels offset most of it, and the totals are 0.02 and -0.07.

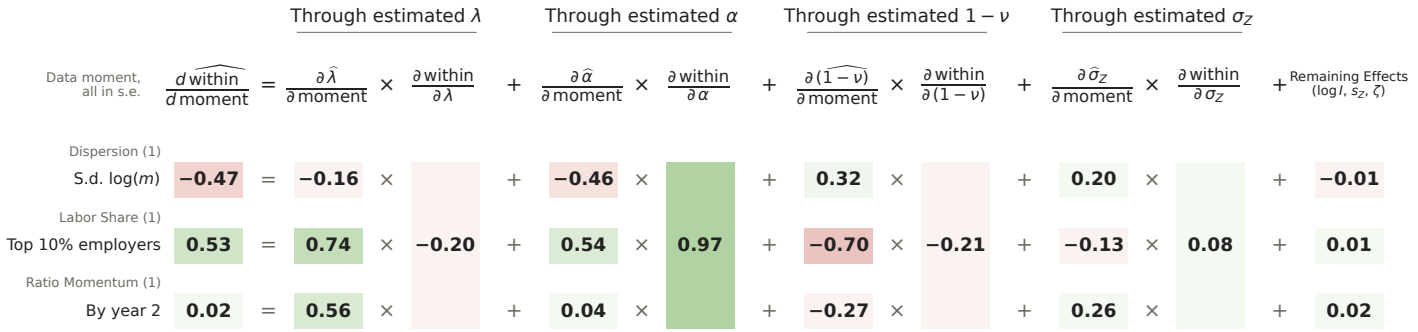
A lower weight on managers in production (i.e., smaller α) reduces the share of value from reuse within partitions. The need to reassign investors to a new manager is smaller because output depends less on managers. Firms derive more value from reuse across partitions when this weight is lower. This is because reuse across partitions lets the firm repair imbalances with its existing members and thus reduces the need to suffer search costs. These costs are especially large at low α because the market utilities of investors and managers rise in equilibrium (Figure 4).

A larger labor share of income (among the top 10% of employers) also suggests higher α. Through α, a one-standard-error rise in this labor share raises the share of value from reuse within partitions by 0.52 standard errors (i.e., 0.54×0.97), of a total of 0.53. It lowers the share of value from reuse across partitions by 0.44 (i.e., 0.54×0.80), of a total of 0.59.

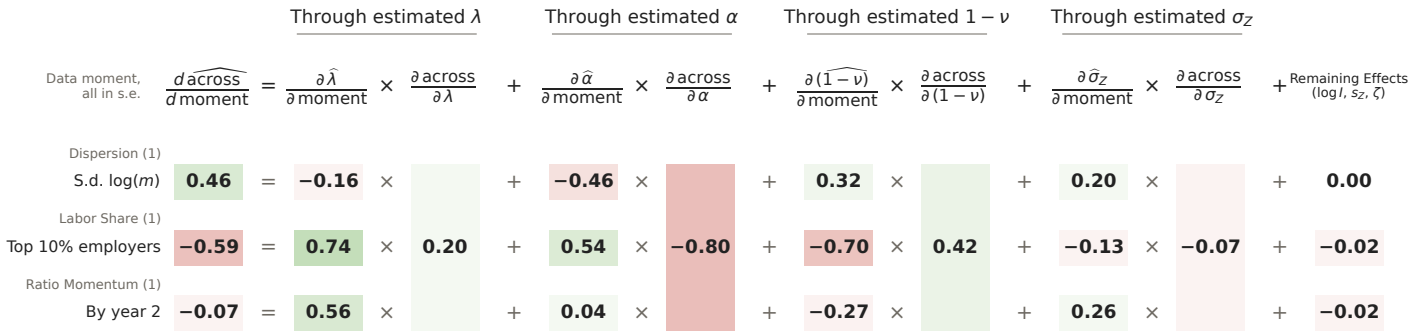
[25] For each share, I find the three targeted moments that raise its asymptotic standard error the most when removed jointly from the estimation (Honoré et al., 2020). The two shares have the same three moments. The four parameters carry the largest channels, in absolute value, over these moments and both shares.

Figure 12: Sensitivity to Moments and Parameters

(a) Share of value from reuse within partitions



(b) Share of value from reuse across partitions

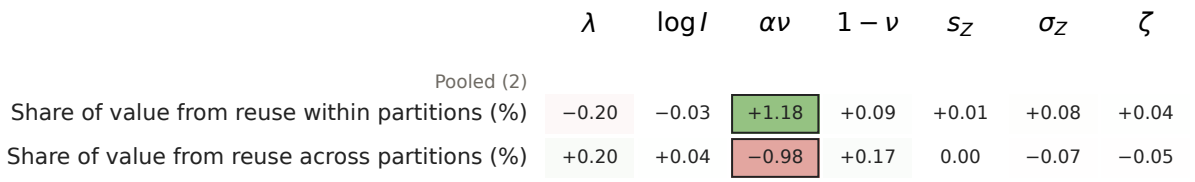


Note: Rows: each share's three targeted moments that raise its standard error the most when removed jointly from the estimation (Honoré et al., 2020), which are the standard deviation of $\log(m)$, the labor share, and the ratio momentum by year 2 for both shares. All quantities are in their own standard errors. $\partial\widehat{\Theta}_k/\partial\widehat{\psi}_j$ is the sensitivity of the estimator of Section 4.4 to the data moment (Andrews et al., 2017). The share's derivative in each parameter is at the estimates, in the new equilibrium. Every parameter: Figures 5 and 13.

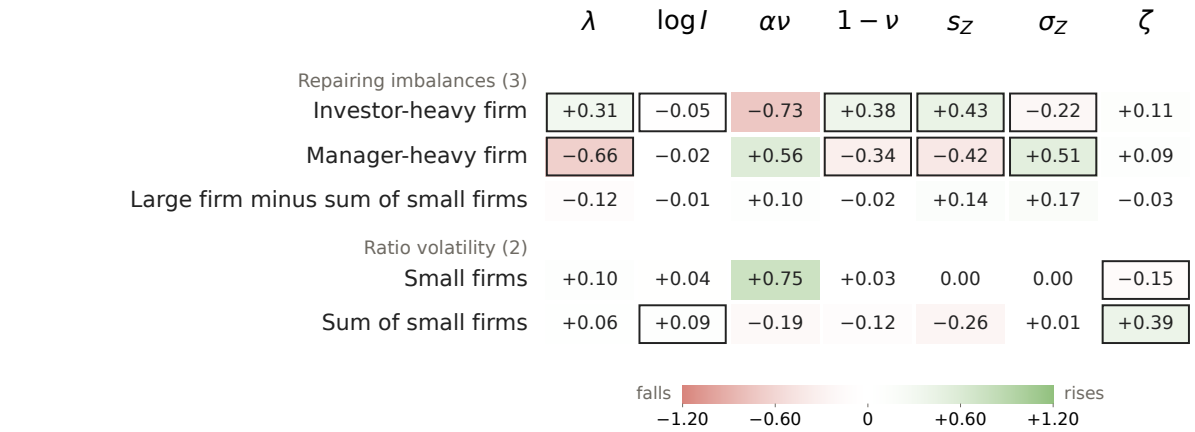
Among all targeted moments, the year-1 investor leg moves both shares the most (Figure 13). It raises the share of value from reuse within partitions by 0.79 standard errors and lowers the share of value from reuse across partitions by 0.78.

Figure 13: Sensitivity to All Parameters

(a) Value Decomposition



(b) Untargeted Moments



Note: Panel (a) as in Figure 12, for every estimated parameter. In panel (b), each cell is the change in an untargeted moment of Figure 1, in its data standard errors, when the parameter rises by one of its standard errors, in the new equilibrium. The two panels share one color scale. Framed: each parameter's two largest cells across both panels.

E.2 Value of a Partition without an Internal Market

A partition in Definition 4 has one manager, k investors, and its share of the firm's Z and the implied z, and it has no internal market. Its value $w(k, z)$ solves

$$\begin{aligned} r\, w(k,z) = \max_{\mu_I \geq 0}\; & z\tilde{f}(k)^{\nu} - k(r+\epsilon_I)U_I - (r+\epsilon_M)U_M \\ & + \mu_I \left[w(k+1,z) - w(k,z) - \bar{R}(\mu_I; U_I)\right] \\ & + k\,\epsilon_I \left[w(k-1,z) - w(k,z)\right] \qquad \text{(Partition's HJB)} \\ & + \epsilon_M \left[0 - w(k,z)\right] \\ & + \mu_z\, z\, \frac{\partial w(k,z)}{\partial z} + \frac{\sigma_z^2}{2}\, z^2\, \frac{\partial^2 w(k,z)}{\partial z^2}, \end{aligned}$$

where μ_z and σ_z are as in equation (Firm's HJB with Z). Free disposal lets the partition release investors or close at no cost. Hence $w(k, z) \geq w(k', z)$ for every $k' < k$ and $w(k, z) \geq 0$, and equation (Partition's HJB) holds wherever the partition keeps all its members.

Equation (Partition's HJB) is equation (Firm's HJB) at $m = 1$, extended with z and free disposal as in the estimated model, with two changes. First, the partition does not recruit managers. Without an internal market, the partition cannot assign its investors or its Z to a new manager, who would produce nothing. A firm $(k, 1, z)$ can instead recruit a manager and move to $(k, 2, z)$. Second, the partition closes when its manager departs, and its investors return to search. A firm $(k, 1, z)$ instead moves to $(k, 0, z)$, keeps its investors, and recruits a manager later. Therefore, a partition grows only in investors.

Each partition starts with its own z_j. At fixed U_I and U_M, each partition's flow payoff, recruiting costs, and departures depend only on its own state. Hence the value of the m partitions is $\sum_{j=1}^{m} w(k_j^*, z_j)$.